Elliptical Orbits

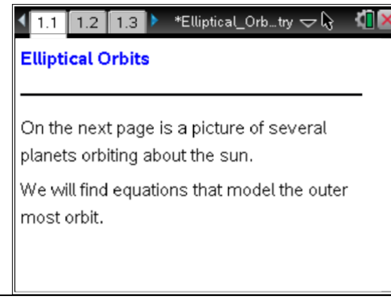
Student Activity

Name ______________

Class ______________

Open the TI-Nspire™ document *eliiptical_orbits.tns.*

In this activity, you will be given a photograph of elliptical orbits embedded in a graph screen, and you will model the path of one orbit using equations of ellipses.



Move to page 1.2.

Press [ctrl] ▶ and [ctrl] ◀ to navigate through the lesson.

We are going to model the path of the largest orbit by using the equation of an ellipse.

1. Grab & move the four points near the origin and place the points at each of the four vertices of the ellipse. Notice that the horizontal vertices are **not** on the x-axis, while the vertical vertices *are* on the y-axis. (To grab a point, press [ctrl] [click] and use the Touchpad arrows to move. Press [esc] to release the point when the ordered pairs show.)

Note: If you make a mistake, or are not happy with where the point is placed, immediately press [ctrl] [esc] to undo the last action.

For reference purposes, we are going to call the four points at the vertices P, Q, R, and S. P is the top vertical point; R is the bottom vertical point. S is the horizontal point on the left side, and Q is the horizontal point on the right side.

2. Write the coordinates in the spaces provided. Include all digits shown on your TI-Nspire handheld screen.

P: (____0____, ________)
Px Py

S: (_________, _________)
Sx Sy

Q: (_________, _________)
Qx Qy

R: (____0_____, ________)
Rx Ry

Note: Px stands for the x-coordinate of point P, Py stands for the y-coordinate of point P, and so on.

Move to page 1.3.

Press [ctrl] ▶ and [ctrl] ◀ to navigate through the lesson.

Note: This is a calculator page. Use this page for any calculations that need to be performed. Record the results on this worksheet in the appropriate spaces provided.

The center of the ellipse should be close to the midpoints of segments SQ and PR. To approximate the center, we will calculate the *x*-coordinate of the midpoint of segment SQ, and the *y*-coordinate of the midpoint of segment PR.

3. To find the *x*-coordinate of the midpoint of segment SQ, find the average of the x-coordinates:

$$\frac{Sx+Qx}{2} =$$

4. To find the *y*-coordinate of the midpoint of segment PR, find the average of the *y*-coordinates:

$$\frac{Py+Ry}{2} =$$

5. Round your final answers to the nearest tenth. The coordinates of the center of the ellipse are approximately:

(____________, ____________)
h k

The general equation of an ellipse is: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The value for a is half the length of segment SQ. The value for b is half the length of segment PR.

6. Calculate those two values on the worksheet below. Use page 1.3 (or the Scratchpad) of the TI-Nspire document to do any computations. Do not round any final answers until told to do so.

a = half the length of segment SQ = $\frac{|Qx-Sx|}{2}$

b = half the length of segment PR = $\frac{|Py-Ry|}{2}$

7. Since the equation of the ellipse squares the values for both a and b, square each of the values, and record your answers below:

a^2 = ___________ b^2 = ___________

8. For our purposes, we will round those values to the nearest tenth:

a^2 = ___________ (to the nearest tenth) b^2 = ___________ (to the nearest tenth)

9. Substitute the values of h, k, a^2, and b^2 into $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$. This is the general equation of an ellipse whose center is at (h, k). Write your equation here:

10. Solve for y, and record your work next to each following step:

   a. First, clear out the denominators by multiplying each term by $a^2 \cdot b^2$. The result should be:
   $$b^2 \cdot (x-h)^2 + a^2 \cdot (y-k)^2 = a^2 \cdot b^2$$

   b. Subtract the first term (the one involving x) from each side:
   $$a^2 \cdot (y-k)^2 = a^2 \cdot b^2 - b^2 \cdot (x-h)^2$$

   c. Divide each term by a^2:
   $$(y-k)^2 = b^2 - \frac{b^2}{a^2} \cdot (x-h)^2$$

   d. Take the square root of each side.
   $$y-k = \pm\sqrt{b^2 - \frac{b^2}{a^2}(x-h)^2}$$
   (make sure that you have both roots: $\pm$)

   e. Add k to each side to finish:
   $$y = k \pm \sqrt{b^2 - \frac{b^2}{a^2}(x-h)^2}$$

Move back to page 1.2.

Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

11. Press ctrl G to make the function input line appear, type one of the equations you just obtained into $f1(x)$, and press enter.

For example: $f1(x) = k + \sqrt{b^2 - \frac{b^2}{a^2}(x-h)^2}$

The top half of the ellipse should closely approximate the top half of the path of the planet. If so, you did all the algebra correctly. If the ellipse does not follow the path of the planet, go back, and recheck your algebra.

12. Press ctrl G to make the function input line appear again, and type the other equation you obtained into $f2(x)$, and press enter.