Special Segments in Triangles

About the Lesson

In this activity, students will construct and explore medians, altitudes, angle bisectors, and perpendicular bisectors of triangles. They then drag the vertices to see where the intersections of the segments lie in relation to the triangle and measure distances to identify relationships. As a result, students will:

- Construct the angle bisectors of a triangle and drag a vertex to discover that they are concurrent.
- Construct the perpendicular bisectors of a triangle and drag a vertex to discover that they are concurrent.
- Construct the altitudes of a triangle and postulate a theorem about the properties of its altitudes.
- Construct the medians of a triangle and postulate a theorem about the properties of its medians.
- Explore properties of the four points of concurrency (incenter, circumcenter, orthocenter, centroid).

Vocabulary

- incenter
- orthocenter
- circumcenter
- centroid

Teacher Preparation and Notes

- Before starting this activity, students should be familiar with the following terms: angle bisector, perpendicular bisector, altitude, median, and point of concurrency.
- Problem 1 can be extended by having students construct the circle inscribed in the triangle, where a radius can be drawn by constructing a line through the incenter perpendicular to a side of the triangle. Problem 2 can be extended by having students construct the circle circumscribed about the triangle, where a radius can be drawn by constructing a line segment from the circumcenter to any one of the vertices of the triangle.
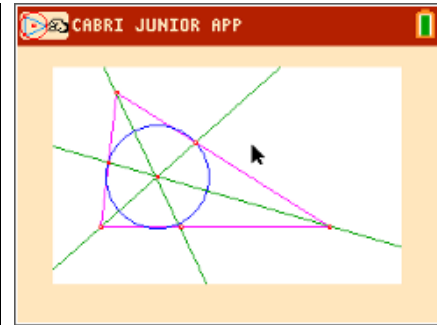
Activity Materials

- Compatible TI Technologies:

    TI-84 Plus*
    TI-84 Plus Silver Edition*
    🌈TI-84 Plus C Silver Edition
    🌈TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint^{TM} functionality.

Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.
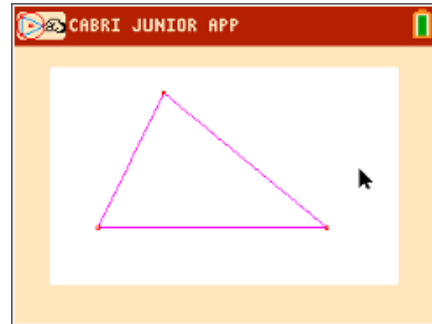
Lesson Files:

- Special_Segments_in_Triangles _Student.pdf
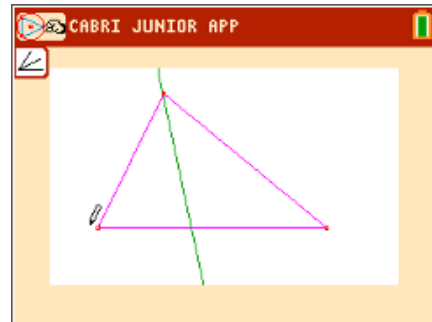- Special_Segments_in_Triangles _Student.doc

Problem 1 – Angle Bisectors & the Incenter

Students should open a new Cabri Jr. file. With the **Triangle** tool, construct a triangle.

Students are to construct the angle bisector of one angle using the **Angle Bis.** tool.

With the **Angle Bis.** tool still active, construct the angle bisector of the other two angles of the triangle. Press clear to exit the tool.

Direct students to drag a vertex of the triangle and observe the results.

Students should see that the three angle bisectors of the triangle intersect each other at one point.
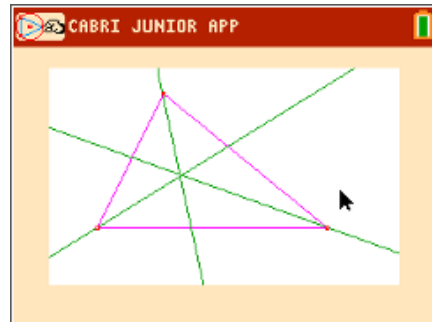
Have students place a point at the intersection of the angle bisectors using the **Point>Intersection** tool. This point of concurrency is called the incenter.

![TI logo] **Special Segments in Triangles**

TEACHER NOTES
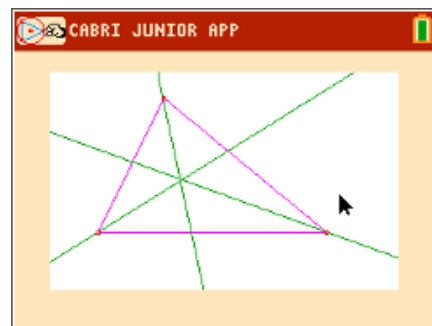
Drag a vertex of the triangle to determine the location of the incenter in relation to the triangle and record the results on the worksheet.

Students should examine acute, right, and obtuse triangles. They will see that the incenter is always located inside of the triangle.
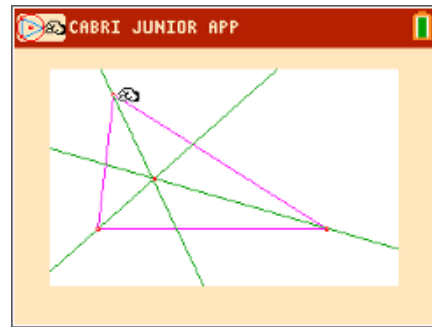
1. What appears to be true about the three angle bisectors of the triangle?

   **Sample Answer:** The incenter is always located inside of the triangle.

Students need to use the **Point>Intersection** tool to place a point where an angle bisector intersects a side of the triangle.

Students are then to use the **Circle** tool to draw a circle from the incenter to the point on the side of the triangle.
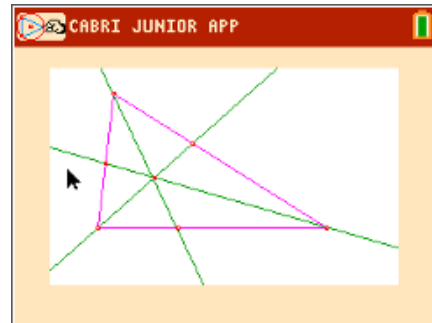
They will see that this circle is inscribed in the triangle; that is, the circle just "touches" each side of the triangle.

Students can save this file as *Incenter*.
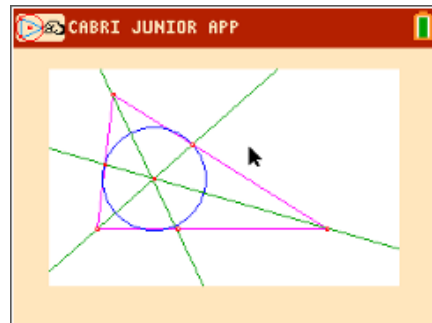
2. In relation to the triangle, where is the incenter located?

   Acute Triangle: _____

   Right Triangle: _____

   Obtuse Triangle: _____

   Answer: For all 3 triangles: inside of the triangle at the intersection of the angle bisectors

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Problem 2 – Perpendicular Bisectors & the Circumcenter

Students should open a new Cabri Jr. file. With the **Triangle** tool, construct a triangle.

Students are to construct the perpendicular bisector of one side using the **Perp. Bis.** tool.
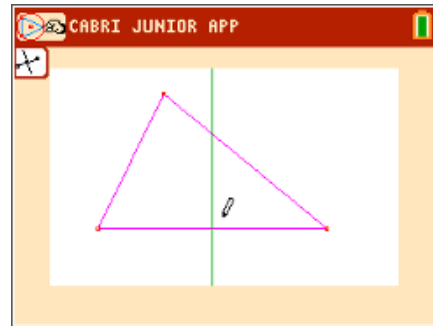
With the **Perp. Bis.** tool still active, construct the perpendicular bisector of the other two sides of the triangle. Press clear to exit the tool.

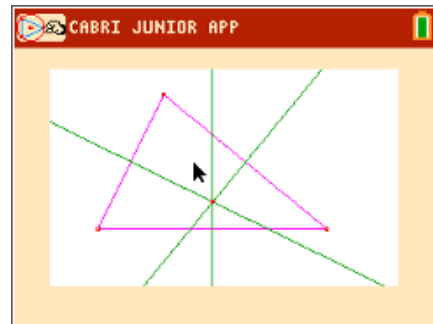Direct students to drag a vertex of the triangle and observe the results.

Students should see that the three perpendicular bisectors of the triangle intersect each other at one point.
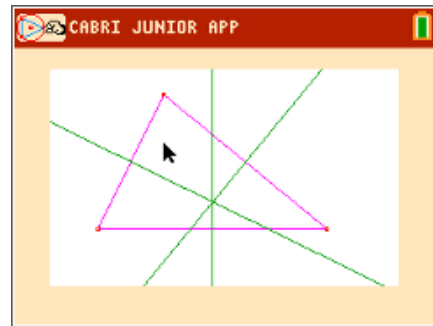
3. What appears to be true about the three perpendicular bisectors of the triangle?

> **Sample Answer:** The three perpendicular bisectors of the triangle intersect each other at one point.

Have students place a point at the intersection of the perpendicular bisectors using the **Point>Intersection** tool. This point of concurrency is called the circumcenter.

Drag a vertex of the triangle to determine the location of the circumcenter in relation to the triangle and record the results on the worksheet.

Students should examine acute, right, and obtuse triangles. They will see that the circumcenter lies inside the triangle if the triangle is acute, outside the triangle if it is obtuse, and on the triangle if it is right.
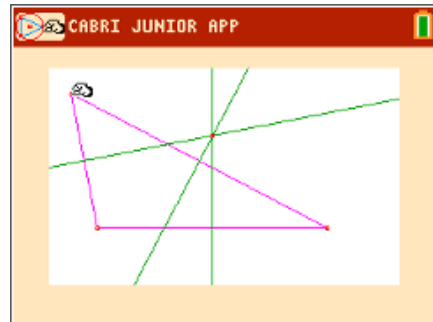
4. In relation to the triangle, where is the circumcenter located?
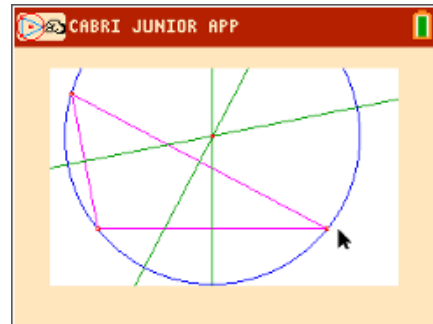
Acute Triangle: _____

Right Triangle: _____

Obtuse Triangle: _____

Answer: inside the triangle; outside the triangle; on the triangle

Students are to use the **Circle** tool to draw a circle from the circumcenter to a vertex of the triangle. They will see that this circle is circumscribed about the triangle; that is, each vertex of the triangle lies on the circle.
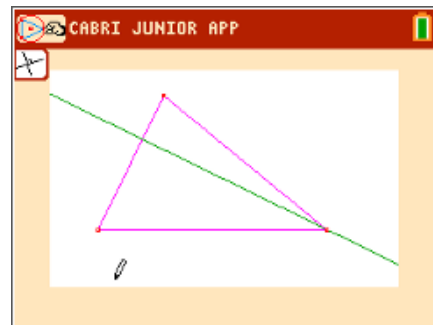
Students can save this file as *Circumce*.

Problem 3 – Altitudes & the Orthocenter

Students should open a new Cabri Jr. file. With the **Triangle** tool, construct a triangle.

Students are to construct one altitude in the triangle using the **Perp.** tool. Select one vertex and select the side opposite that vertex.

With the **Perp.** tool still active, they need to construct the other two altitudes of the triangle. Press clear to exit the tool.

Direct students to drag a vertex of the triangle and observe the results.

Students should see that the three altitudes of the triangle intersect each other at one point.
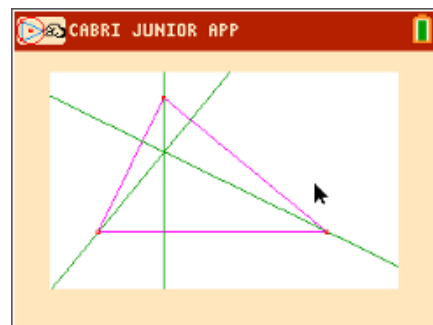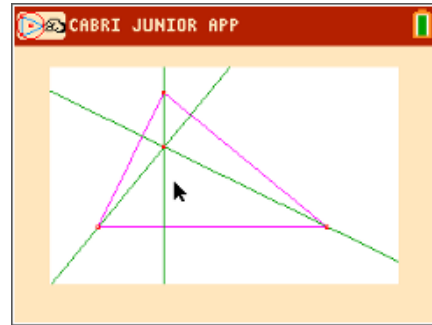
5. What appears to be true about the three altitudes of the triangle?

**Answer:** The three altitudes of the triangle intersect each other at one point.

Students are to place a point at the intersection of the altitudes using the **Point>Intersection** tool. This point of concurrency is called the orthocenter.

Drag a vertex of the triangle to determine the location of the orthocenter and record the results on the worksheet.

Students should examine acute, right, and obtuse triangles. They will see that the orthocenter lies inside the triangle if the triangle is acute, outside the triangle if it is obtuse, and on the triangle if it is right.

Students can save this file as *Orthocen*.
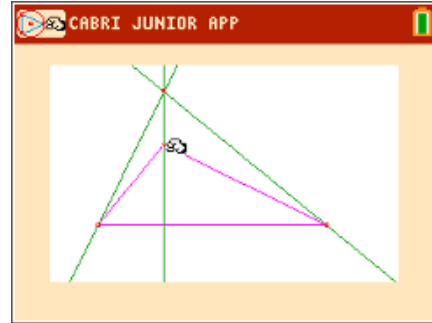
6. In relation to the triangle, where is the orthocenter located?

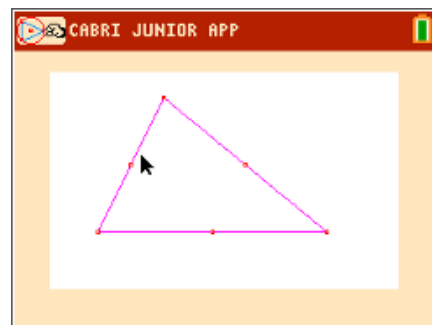Acute Triangle: _____

Right Triangle: _____

Obtuse Triangle: _____

Answer: inside the triangle; outside the triangle; on the triangle

Problem 4 – Medians & the Centroid

Students should open a new Cabri Jr. file. With the **Triangle** tool, construct a triangle.

Students are to construct the midpoints of all three sides of the triangle using the **Midpoint** tool.
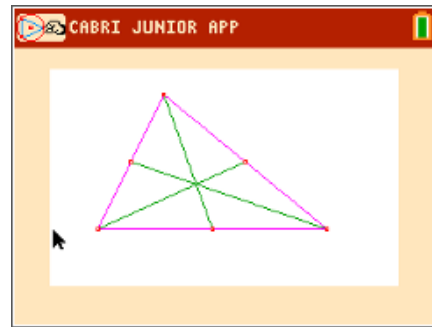
Use the **Segment** tool to connect each vertex to the midpoint of the opposite side.

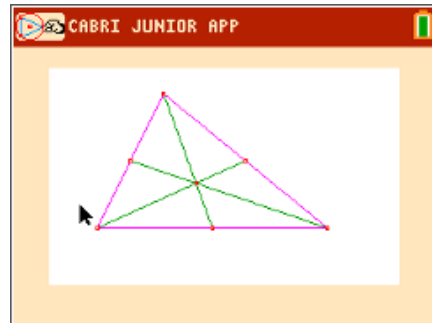Direct students to drag a vertex of the triangle and observe the results.

Students should see that the three medians of the triangle intersect each other at one point.

**7.** What appears to be true about the three medians of the triangle?

Answer: The three medians of the triangle intersect each other at one point.

Have students place a point at the intersection of the medians using the **Point>Intersection** tool. This point of concurrency is called the centroid.

Drag a vertex of the triangle to determine the location of the centroid in relation to the triangle and record the results on the worksheet.

Students should examine acute, right, and obtuse triangles. They will see that the centroid is always located inside of the triangle.
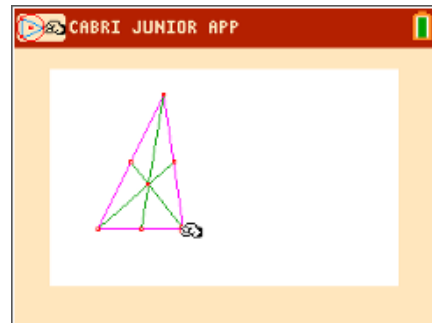
8. In relation to the triangle, where is the centroid located?

**Answer:** The centroid is always located inside of the triangle.

The centroid divides each median into two unequal parts: the segment from the vertex to the centroid, and the segment from the centroid to the midpoint of the opposite side.

To investigate the lengths of these two parts, students are to use the **Measure>D. & Length** tool to measure these on one median.

If desired, hide the other two medians.

Students can drag a vertex of the triangle and observe the lengths changing. They should record some of the values on the worksheet.
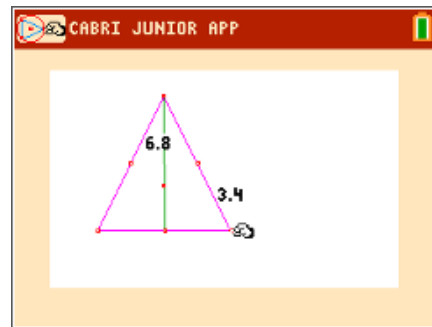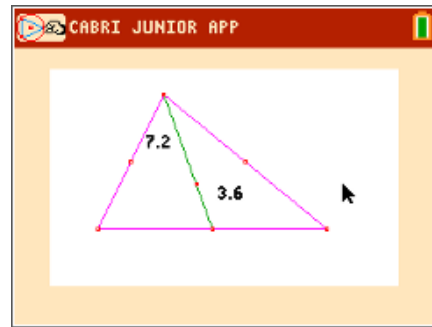
9. Record some measurements from Problem 4.

Sample Answers:

Distance from Vertex to Centroid	Distance from Centroid to Midpoint
7.2	3.6
6.8	3.4
5.4	2.7

Students are to use the **Calculate** tool to find the ratio of the two lengths.

Then they can drag a vertex and observe the results. They should see that the ratio of the lengths from the centroid is always 2.

Students can save this file as *Centroid.*
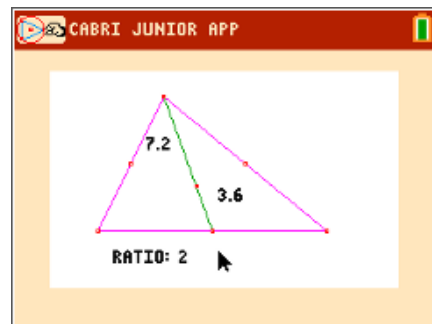
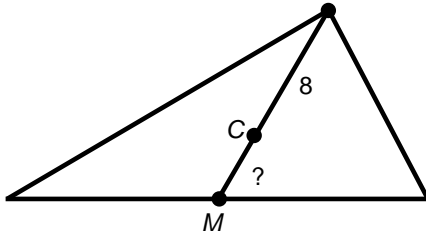**10.** What is the ratio of the two distances?

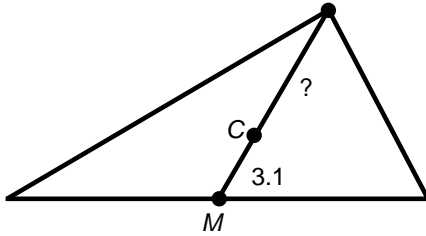**Answer:** The ratio of the lengths from the centroid is always 2.

Exercises

Use your ratio from Question 10 above to find the missing values for each. *C* is the centroid and *M* is a midpoint.

11.

Answer: 4

12.

Answer: 6.2