Consul Educated Monkey

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Teacher Notes and Answers

7   8   9   **10**   11   12

TI-Nspire   Investigation   Student   40 min

Investigation

Consul the Educated Monkey is a mechanical toy that was invented in 1916 to help students learn their multiplication tables. Students move the monkey's feet to select two integer values and the monkey's hands point to the answer. The geometrically designed toy can be used for more than multiplication. In this investigation you will determine how and why the device works and design further applications.

Teacher Notes:

Where did the name Consul come from? A 1909 vaudeville film titled: "Consul crosses the Atlantic" featured a performing monkey based on Consul the monkey that performed in "Wombell's Travelling Manageries" in 1893. Consul was later sold to Belle Vue Zoological Gardens. Consul wore clothes, smoked a pipe and performed a range of human like actions. The original Consul died in 1894 but was soon replaced by Consul Junior with the name "Consul" becoming synonymous with any monkey portraying human like characteristics. Interestingly the original toy (1916), "Consul the Educated Monkey" included an interesting sales line on the packaging: "It makes no difference to Monkey whether children are bright or stupid, he never loses patience at having to answer their questions".

Open the TI-Nspire file: Consul Educated Monkey

Navigate to Page 1.2. Use the navigation pad to move the mouse over either point A or Point B corresponding to Consul's feet. When the mouse changes to an open-hand, click and hold (or press Ctrl and Click) to grab the monkey's feet. Move the feet left / right to check the accuracy of the model and observe how the linkages move.

Note: In the physical version Consul's legs cannot touch or cross.
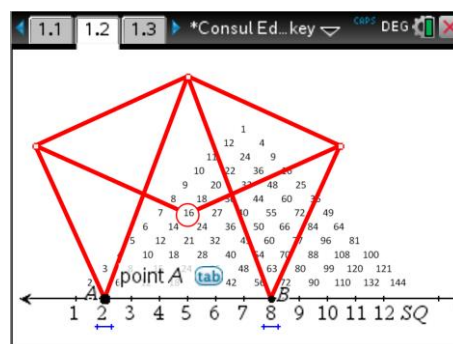
Question: 1.

Place Consul's left leg on 3. Move Consul's right leg from 4 through to 12. What do you notice about the movement of the circle that highlights the answer to each multiplication problem?

The circle, Consul's hands; appear to move in a straight line. [Appear – Required to prove later].

Question: 2.

Place Consul's right leg on a number greater than 8, then move Consul's left leg. What do you notice about the movement of the circle that highlights the answer?

The circle, Consul's hands; appear to move in a straight line. [Appear – Required to prove later.]

Question: 3.

Can the entire multiplication tables 1×1 through to 12×12 be generated using Consul?

Yes, provided the user understands the commutative properties that $2 \times 3 = 3 \times 2$ and that perfect squares are produced under the squared section: SQ (far right).

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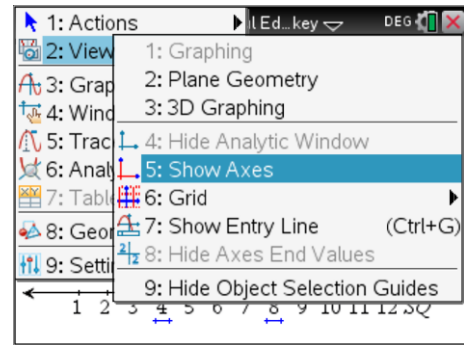
TEXAS INSTRUMENTS

How does it work – Part 1

Navigate to page 2.1.

The Consul construction is available minus the highlight circle and background numbers. A set of Axis can be used to track the location of point P (Consul's hands).

Move point A to (1, 0).

Grab point B and move it along the x axis. Write down the coordinates of point P for each location of point B.

1: Actions	▶	l Ed…key ⌄	DEG ◖▮▸ ✕
🖼 2: View	1: Graphing		
♣ 3: Grap	2: Plane Geometry		
🖥 4: Wind	3: 3D Graphing		
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	9: Hide Object Selection Guides		
←	1 2 3 4 5 6 7 8 9 10 11 12 ⌄Q		

Question: 4.

Record the location of point B and corresponding location of point P in the table below.   Point A = (1, 0)
In the table below P_x = x coordinate of point P and P_y = y coordinate of point P. Determine an equation relating (P_x , P_y) and graph your equation. Once again, drag point B to ensure the equation is correct.

Point B	2	3	4	5	6	7	8	9	10	11	12
P_x	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5
P_y	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5

Equation: $y = x - 1$

Question: 5.

Set Point A at (2, 0). Determine the equation for the new line passing through the set of points: (P_x, P_y). Graph your equation and drag point B to ensure the equation is correct.

Point B	3	4	5	6	7	8	9	10	11	12
P_x	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
P_y	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0

Equation: $y = x - 2$

Question: 6.

Suppose point A is at (5, 0), without constructing a table of values, what do you think the equation might be relating point (P_x, P_y). Check your answer by graphing it and moving point B accordingly.
Equation: $y = x - 5$

Question: 7.

Explore how point P moves when point B is located at (10, 0) and point A is moved. Determine the corresponding motion point P.
Equation: $y = 10 - x$

Author: P.Fox

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How does it work – Part 2

So far it would appear that point P moves in a straight line. We can prove this by knowing how the linkages in Consul are constructed and combine a little geometric reasoning.
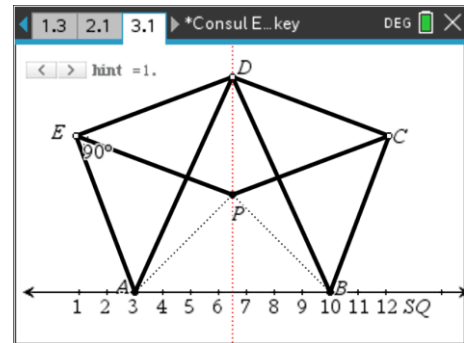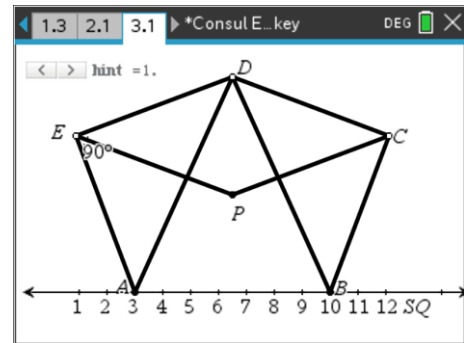
Here are a few things about the physical construction:

$\angle$ AED = $\angle$ BCD = 90°

Distance: AE = EP = ED = BC = CP = CD

Consul is symmetrical about the line passing through points D and P.

In this first stage it will be useful to prove that $\angle$APB = 90°.

The questions below provide a guide as to how to proceed with such a proof. The TI-Nspire file can also be used to step through the hints.

Question: 8.

a) What type of triangle is $\triangle$ APE?
Answer: Isosceles since EA = EP. (Given conditions)

b) Write a relationship between $\angle$EAP and $\angle$APE.
Answer: $\angle$EAP = $\angle$APE (Isosceles triangle)

c) What type of triangle is $\triangle$EPD?
Answer: Isosceles since ED = EP. (Given conditions)

d) Write a relationship between $\angle$EPD and $\angle$PDE.
Answer: $\angle$EPD = $\angle$PDE (Isosceles triangle)

e) Determine the value of $\angle$APE + $\angle$EPD. [Note $\angle$APE + $\angle$EPD = $\angle$APD]
Answer: There are two ways this can be done. One way is to consider $\triangle$APE and $\triangle$EPD, another is to consider the quadrilateral AEDP, both methods result in the same relationship:
$\angle$EAP + 90° + $\angle$PDE + $\angle$EPD + $\angle$ APE = 360°

2 x ($\angle$APE + $\angle$EPD) = 270°   [From Part B & D]

$\angle$APE + $\angle$EPD = 135°

f) Given Consul is symmetrical, determine the value of $\angle$APB.
Answer: $\angle$ APB = 90° .
Students could work through the above or simply state "by symmetry" $\angle$BPD = 135°, therefore $\angle$APD + $\angle$DPB = 270, therefore $\angle$ APB = 90°.

Question: 9.

Given the symmetry of Consul and $\angle$APB = 90°, determine $\angle$BAP.
Answer: $\triangle$BAP is an isosceles triangle (symmetry) and $\angle$APB = 90°, therefore $\angle$BAP = 45°.

Author: P.Fox

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Question: 10.

Let point Q be the midpoint of AB. If length AQ is doubled, what happens to length QP? (Explain)

Answer: If AQ is doubled then QP will be doubled as $\triangle$ APB is isosceles.

Question: 11.

Suppose Consul is superimposed onto the Cartesian plane (as per Page 2.1 of the TI-nspire document), what does your answer to Question 10 say about the calculation: $\dfrac{\Delta y}{\Delta x}$ and hence the movement of point P.

Answer: $\dfrac{\Delta y}{\Delta x} = \text{constant}$ therefore P will move in a straight line (constant gradient), furthermore the gradient of the line will be either +1 or -1 depending on the respective changes in x and y.

Question: 12.

Use your answers to help explain how Consul works and show how Consul could be used for basic addition.

Answer: Point P moves in a straight line when either A OR B is moved (not both), furthermore as either A or B is moved the straight lines have the same gradient and therefore will not cross. This movement means that for each position of A or B a unique position for point P exists. This unique position means that a grid can be constructed representing the products of A and B.

Note that the 'SQ' reference would become '13'.

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                        14
                     13    15
                  12    14    16
               11    13    15    17
            10    12    14    16    18
          9    11    13    15    17    19
        8    10    12    14    16    18    20
       7    9    11    13    15    17    19    21
     6    8    10    12    14    16    18    20    22
    5    7    9    11    13    15    17    19    21    23
   4    6    8    10    12    14    16    18    20    22    24
  3    5    7    9    11    13    15    17    19    21    23    25
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Teacher Notes:
This activity could also be extended to include a table for combinatorics. If the numbers on the base of Consul are reversed: 13, 12, 11, 10 … then it would be possible to evaluate $^{13}C_2$ etc… The numbers would however need to be quite small as $^{13}C_6 = 1716$. How would it relate to Pascal's triangle?

Author: P.Fox

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