What's Normal, Anyway?

ID: 9415

Time required
60 minutes

Activity Overview

In this activity, students explore the normal distribution and several of its most interesting properties. First, they simulate a binomial experiment and use a histogram of the data to examine the general shape of a normal curve. Then, they graph a normal distribution with a given mean and standard deviation. They see how the graph changes when just the mean changes and when just the standard deviation changes. Lastly, they examine the empirical rule for normal distributions, describing the percent of data values falling within different standard deviations from the mean.

Topic: The Normal Distribution

- *Graph the probability density function of the normal distribution for a given mean and standard deviation*
- *Explain how the mean of a random variable is changed under a linear transformation.*
- *Explain how the standard deviation of a random variable is changed under a linear transformation.*

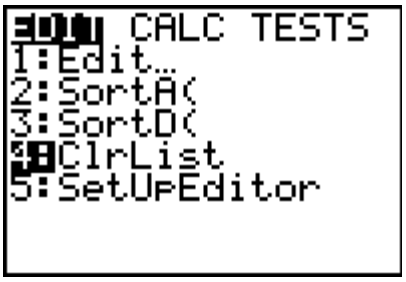
Teacher Preparation and Notes

- *This activity is designed for use in a Precalculus or Statistics classroom. It is best used to ease into the introduction of the normal distribution, after discussion of probability, independent events, and binomial experiments and distributions.*
- *Students should know how to interpret histograms and be familiar with the concepts of mean as a measure of center and standard deviation as a measure of spread.*
- *This activity is designed to have students explore **individually and in pairs**. However, an alternate approach would be to use the activity in a whole-class format. By using the questions found on the student worksheet, you can lead an interactive class discussion.*
- *As students finish Problem 3, discuss how the graph never touches the x-axis and how the remaining 0.3% is divided on either side of the mean beyond three standard deviations. Discuss that all normal distributions follow the 68%-95%-99.5% rule, also known as the Empirical Rule.*
- *At the end, when students reflect back on their percents from Problem 1, have students discuss reasons why their percents do not equal those from the Empirical Rule exactly. For instance, the distribution was not exactly normal, the mean was not necessarily at the center of the histogram, and so on. Increasing the number of trials per experiment would result in a more normal distribution. This can be done on a computer as an extension activity.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9415" in the keyword search box.***

Associated Materials

- *WhatsNormal_Student.doc*

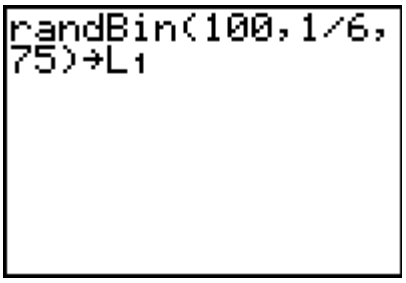
Before beginning this activity, students should be sure that List 1 in the Stat Editor is clear. To clear the list, press **Stat > ClrList > L1**.



Problem 1 – A Binomial Experiment

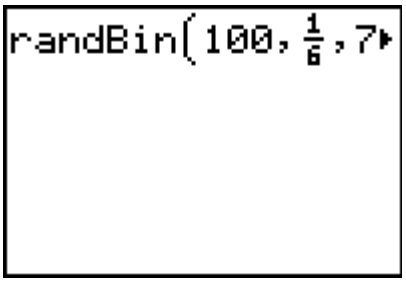
Students begin the activity by simulating 75 experiments of rolling a die 100 times and recording the number of successes where a success is rolling a "3," by doing the following:

Press [MATH], choose **randBin(** from the PRB menu. Type in the number of trials per experiment, the probability of a success, and the number of experiments. Then press [STO▸] and **L1** to store the results in list 1. Press [ENTER].

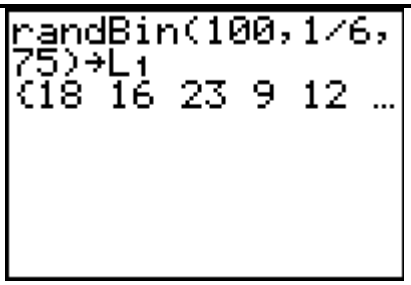


If using Mathprint OS:

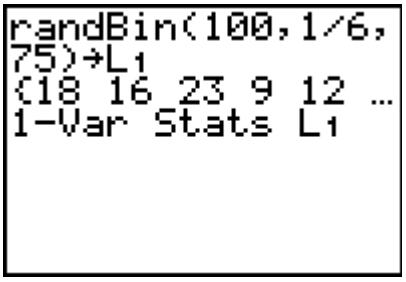
Students can display $\frac{1}{6}$ as a fraction. To do this, enter randBin(100, and then press [ALPHA] [F1] and select **n/d**. Then enter the value of the numerator, **1**, arrow down to the bottom of the fraction and enter the denominator, **6**. Press the right arrow to continue entering the command, **,75)→L1**.



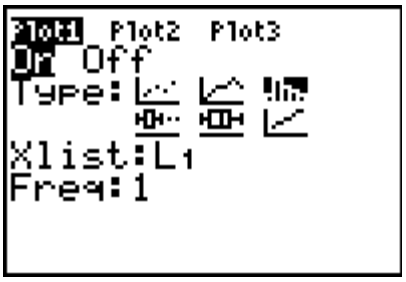
The calculator will take 1-2 minutes to perform the simulations. When it is completed, the beginning of the list of the number of successes per trial will appear. A result of 18 means that out of 100 rolls, a "3" came up 18 times.



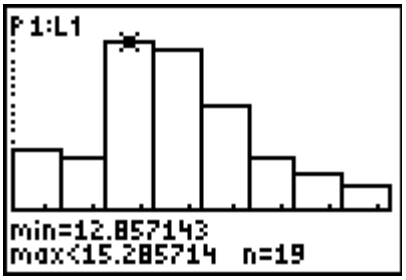
Students select **1-Var Stats** from the CALC menu and then enter **L1** on the home screen. They compute the mean and standard deviation of the data set.



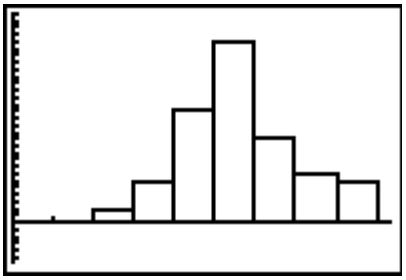
They create a histogram using the settings to the right. To view the histogram, press ZOOM and choose **ZoomStat**.



To find the minimum and maximum of each interval, press TRACE.

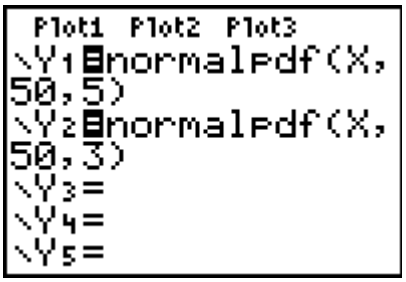


Students should change the scales until all of the bars and the top of each bar are within the viewing window.



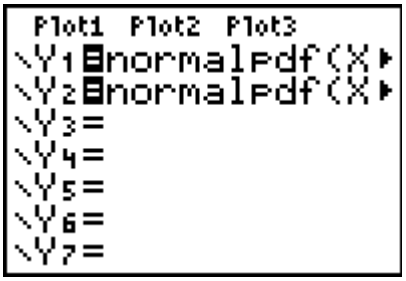
Problem 2 – Properties of the Normal Curve

Students will next explore normal curves and several interesting properties of normal curves. They graph thee normal distribution with a mean of 50 and a standard deviation of 5, a normal distribution with a mean of 50 and a standard deviation of 3, and a normal distribution with a mean of 50 and a standard deviation greater than 5 and compare the graphs. They will conjecture what affects the center and width of the distributions.



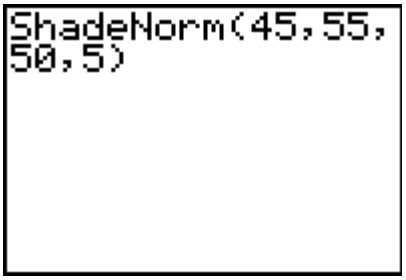
If using Mathprint OS:

When an arrow pointing to the right appears at the end of a function line, the expression continues on. The functions for Y1 and Y2 will run off the screen. To see the entire function, use the right arrow key to scroll to the right.



Problem 3 – The Empirical Rule

Students will explore the area under the normal curve. They discover that 68% of the curve lies within one standard deviation of the mean, 95% lies within two standard deviations of the mean, and 99.7% lies within three standard deviations of the mean.



Solutions

1. $\frac{1}{6}$

2. 16 or 17

3. Sample: 16.4

4. Sample: 4.2

5. Single-peaked, fairly symmetric about the center

6. At or near the center

7. Sample: about 68%, about 95%, about 100%

8. Sample: They're about the same percent. Most of us have 99% or 100% for six intervals.

9. At the center of the graph.

10. It has the same center, but is narrower.

11. It has the same center, but is wider. Sample conjecture: Changing the standard deviation does not change the center of the graph, just the width of the graph.

12. The width is the same, but the whole graph is translated to the left.

13. The width is the same, but the whole graph is translated to the right. Sample conjecture: Changing the mean does not change the shape of the graph, just the location of the graph.

14. The means affects the center; the standard deviation affects the width.

15. 0.682689

16. 68%

17. 95%

18. 99.7%

19. From left to right: 2.35%, 13.5%, 34%, 34%, 13.5%, 2.35%