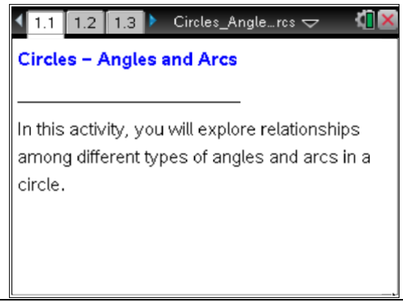
Circles- Angles and Arcs

Student Activity

Name ______________

Class ______________

Open the TI-Nspire document *Circles_Angles_and_Arcs.tns.*



A circle is the set of all points in a plane that are equidistant from a given point in the plane. Circles, angles, and arcs have many interesting characteristics. In this activity, you will explore relationships among different types of angles and arcs in a circle.

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1. Drag point *A* or point *C*. Describe the changes that occur in the figure as you drag the point.

2. Angle *AOC* is called a central angle. Explain why you think this is so.

An angle intercepts an arc of a circle if each endpoint of the arc is on a different ray of the angle and the other points of the arc are in the interior of the angle.

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As you move point *A* or point *C*, the central angle $\angle AOC$ intercepts a minor arc *AC*. The measure of the minor arc equals the measure of the central angle. The larger remaining arc, *ABC*, is called a major arc.

3. a. Move point *A* or point *C* to help you complete the table.

$\angle AOC$	**arc *AC***	**arc *ABC***	**arc *AC* + arc *ABC***
50°	50°		
100°			
		250°	
(Choose an angle.)			

b. With a classmate, discuss what is true about the measure of arc *AC* + arc *ABC*, the sum of the measures of the minor and major arcs. Share your results with the class.

4. In a circle, the measure of a central angle $\angle AOC$ is $n°$.

   a. Find the measure of the minor arc that is intercepted by the central angle. Explain to a classmate how you know this to be true.

   b. Find the measure of the major arc. Explain to a classmate how you know this to be true.

5. Now that you have found the measure of the central angle and the degree measures of both minor and major intercepted arcs, discuss with a classmate how you may be able to find the "length" of those arcs. Discuss the information you would need to find these lengths and the process this would entail.

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6. Angle *ABC* is called an inscribed angle because $\overline{BA}$ and $\overline{BC}$ are chords of the circle and vertex *B* is on the circle. Drag point *B* around the circle.

   a. As point *B* is moved around the circle, discuss with a classmate what you notice about the measure of $\angle ABC$.

   b. With a classmate, discuss why the $m\angle ABC$ changes when point *B* is moved from one arc to the other. Share your reasoning with the class.

c. Move point *A* or point *C* until $\angle ABC$ is a right angle. Discuss with a classmate what is special about the intercepted arc and $\overline{AC}$.

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Angle *ABC* intercepts arc *AC*. Drag point *D* to various locations outside the circle, on the circle, inside the circle, and at the center *O*.

7. Place point *D* on the circle so that $\angle ADC$ intercepts the same arc as $\angle ABC$.
   a. Discuss with a classmate what you notice about the measures of $\angle ABC$ and $\angle ADC$.

   b. Discuss with a classmate what happens to the angles if you move point *A* or point *C*.

8. Place point *D* at the center of the circle, making sure that $\angle ADC$ intercepts the same arc as $\angle ABC$.
   a. Describe the relationship between the measures of inscribed $\angle ABC$ and central $\angle ADC$.

   b. Discuss with a classmate what happens to the angles if you move point *A* or point *C*.

   c. Complete these conjectures:

   - The measure of the inscribed angle is ________________ the measure of the central angle.
   - The measure of the inscribed angle is ________________ the measure of the intercepted arc.

9. Leona said, "Since a central angle can never measure more than 180°, I know an inscribed angle can never measure more than 90°." Discuss with a classmate if you agree or disagree. Explain why.

10. Place point *D* on the circle so that *ABCD* is a quadrilateral.
    a. Discuss with a classmate what you notice about the sum of the measures of ∠*ABC* and ∠*ADC*. Share your results with the class.

    b. Discuss with a classmate what you notice about the sum of the measures of the angles if you move point *A* or point *C*.

    c. Discuss with a classmate what you notice about arcs *ABC* and *ADC*.

    d. Describe how the relationship between arcs *ABC* and *ADC* explain the sum of the measures of inscribed ∠*ABC* and ∠*ADC*.

<u>Further IB Extension</u>

A surveyor standing on the bank of the Reedy River measures the equal distance to the far left end and the far right end of the Liberty Bridge in Greenville, South Carolina, at a central angle of 120°. She found this distance to be 160 ft. See the diagram below (not to scale).

A sector is formed with these two equal distances and the bridge.

(a) Find the arc length of the inner guardrail and the arc length of the outer guardrail.

(b) Find the arc length of the outer guardrail, given that the bridge is a uniform width of 12 ft. across.

(c) Find the area of the walkable portion of the bridge.