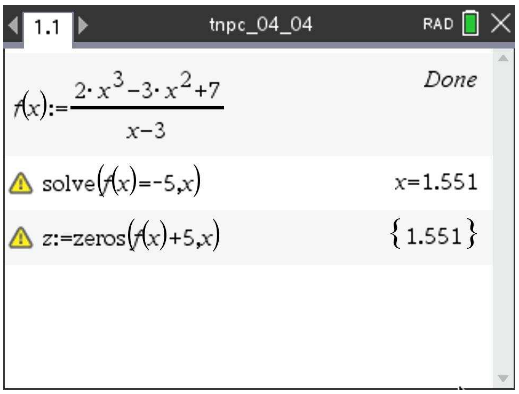
Thursday Night PreCalculus, April 4, 2024

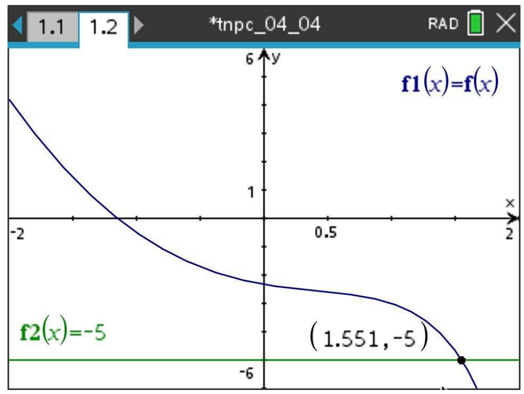
Exam Preparation

Problems

1. The function f is defined by $f(x) = \dfrac{2x^3 - 3x^2 + 7}{x - 3}$. What input value(s) in the domain of f yields an output value of -5?

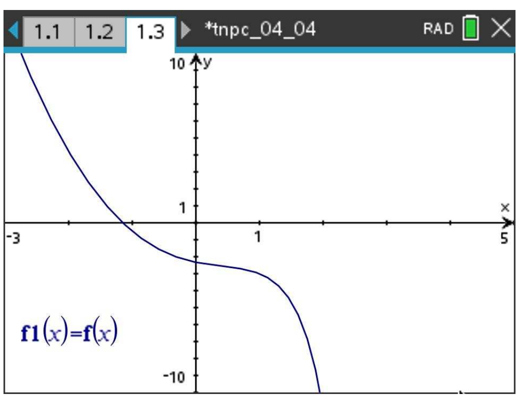
$$f(x) = \frac{2x^3 - 3x^2 + 7}{x - 3} = -5$$

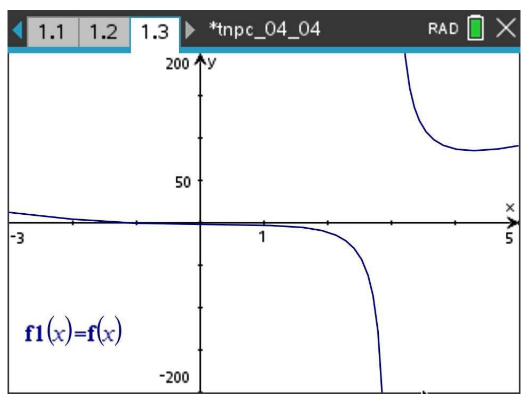




$x = 1.551$ is the input value in the domain of f that yields an output value of -5.

$f(1.551) = -5$

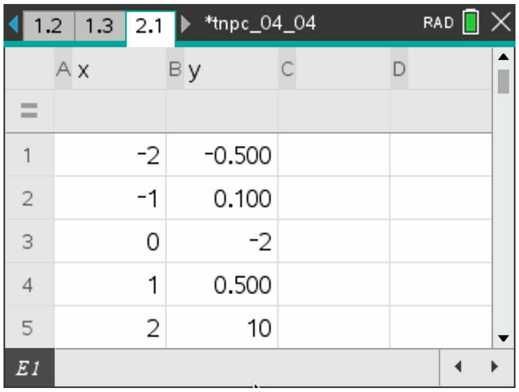


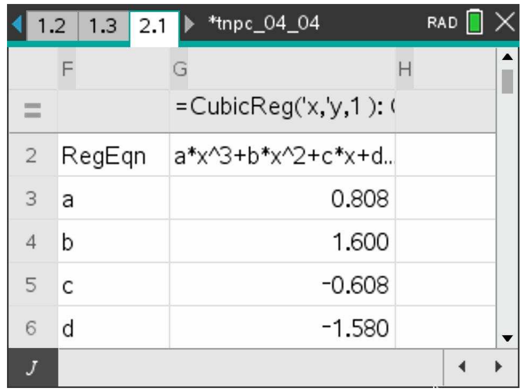


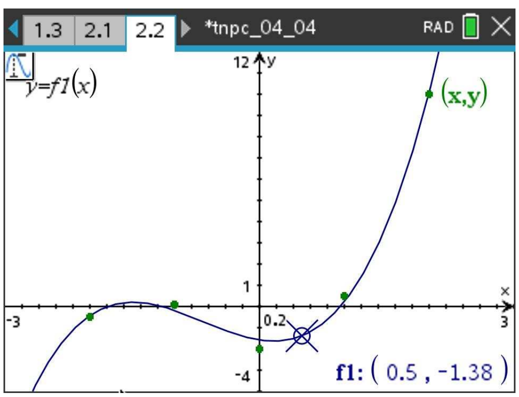
2. The table shows values for a function f at selected values of x.

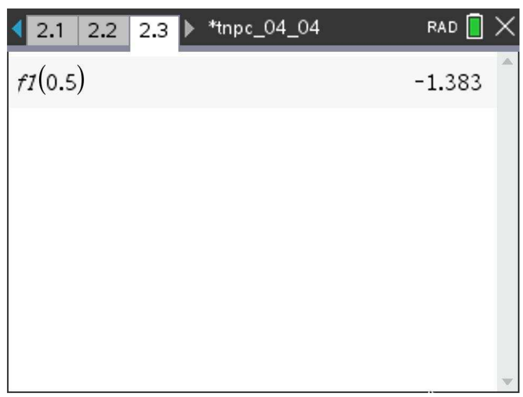
x	-2	-1	0	1	2
$f(x)$	-0.5	0.1	-2	0.5	10

A cubic regression is used to model the function f. What is the value of $f(0.5)$ predicted by the cubic regression model?





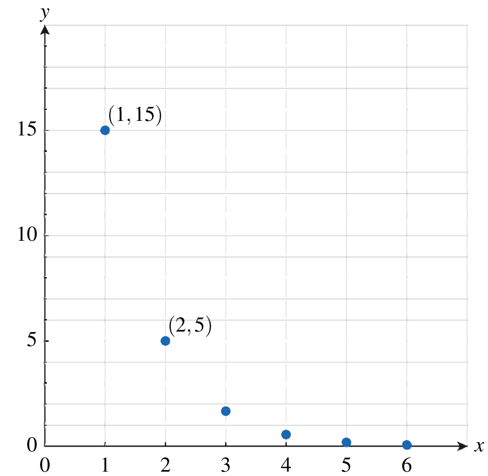




Cubic regression model: $y = 0.808x^3 + 1.6x^2 - 0.608x - 1.58$

Predicted value: $y(0.5) = -1.383$

3. A geometric sequence has the form $g_n = g_k \cdot r^{(n-k)}$. The graph of a geometric sequence , g_n, is shown in the figure.



What is the value of g_5.

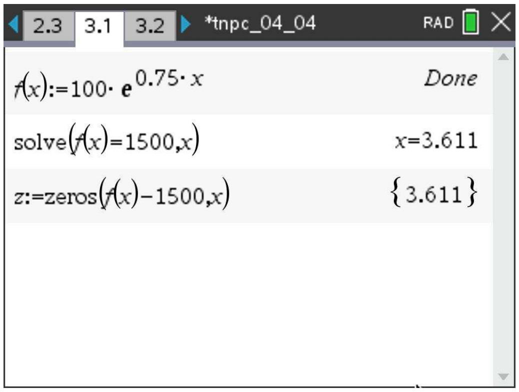
$$g_2 = g_1 \cdot r^{(2-1)} \quad \Rightarrow \quad 5 = 15 \cdot r \quad \Rightarrow \quad r = \frac{1}{3}$$

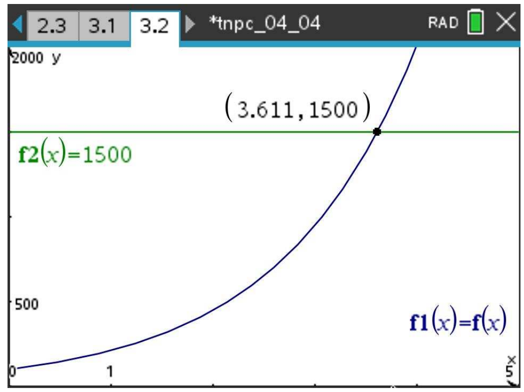
$$g_n = g_1 \cdot \left(\frac{1}{3}\right)^{n-1}$$

$$g_5 = 15 \cdot \left(\frac{1}{3}\right)^{5-1} = 15 \cdot \frac{1}{3^4} = \frac{15}{81} = \frac{5}{27}$$

4. The growth of bacteria in a culture is modeled by $y = 100e^{0.75t}$, where t is measured in days. At what time t is the number of bacteria approximately 1500?

Solve: $100e^{0.75t} = 1500$

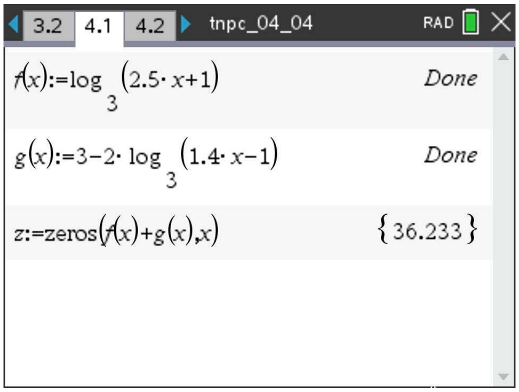


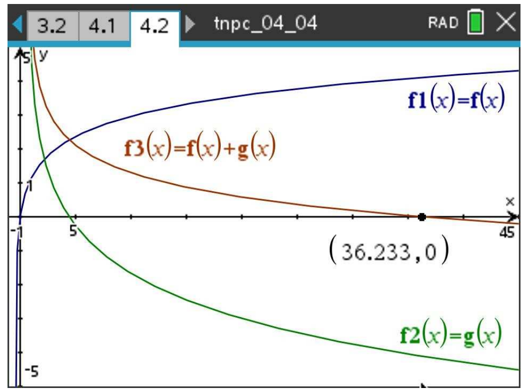


$t = 3.611$ days

5. Consider the logarithmic functions f and g defined by $f(x) = \log_3(2.5x + 1)$ and $g(x) = 3 - 2\log_3(1.4x - 1)$. Find a zero of the function h defined by $h(x) = f(x) + g(x)$.

Solve: $f(x) + g(x) = 0$ for x





$$\log_3(2.5x + 1) = 2\log_3(1.4x - 1) - 3$$

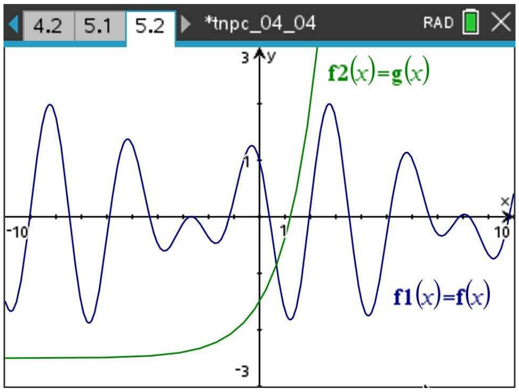
$$2.5x + 1 = (1.4x - 1)^2 \cdot 3^{-3} \quad \Rightarrow \quad 27(2.5x + 1) = 1.96x^2 - 2.8x + 1$$

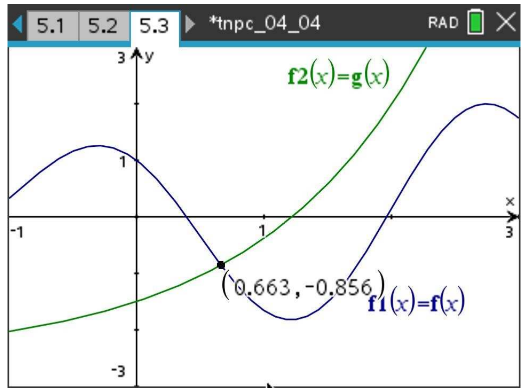
$$67.5x + 27 = 1.96x^2 - 2.8x + 1 \quad \Rightarrow \quad 1.96x^2 - 70.3x - 26 = 0$$

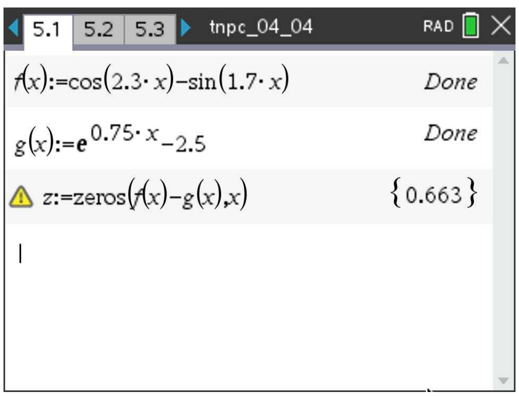
$$x = \frac{70.3 \pm \sqrt{(-70.3)^2 - 4 \cdot 1.96 \cdot (-26)}}{2 \cdot 1.96} = \cdots = -0.366, \quad 36.233$$

Since -0.366 is not in the domain of g, 36.233 is the only zero of $f + g$.

6. The function f is given by $f(x) = \cos(2.3x) - \sin(1.7x)$. The function g is given by $g(x) = e^{0.75x} - 2.5$. Find the input value such that $f(x) = g(x)$.

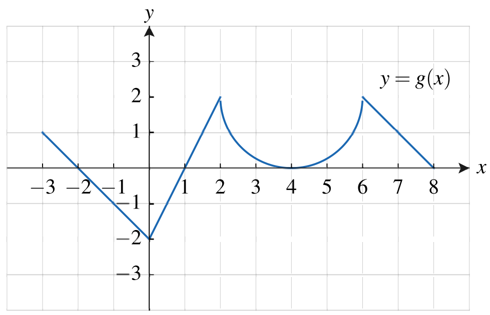






$x = 0.663$

7. The graph of the function g is shown in the figure, and consists of three line segments and a semicircle with radius 2.



The function f is given by $f(x) = \dfrac{-3x^2 + 1.9x + 4.5}{x^3 + 2x^2 + 1}$.

(A) **(i)** The function h is defined by $h(x) = (f \circ g)(x) = f(g(x))$. Find the value of $h(7)$, or indicate that it is not defined.

$$h(7) = (f \circ g)(7) = f(g(7))$$

$$= f(1) = \frac{-3 \cdot 1^2 + 1.9 \cdot 1 + 4.5}{1^3 + 2 \cdot 1^2 + 1}$$

$$= \frac{3.4}{4} = 0.85$$

(ii) Find all values of x for which $g(x) = -1$, or indicate there are no such values.

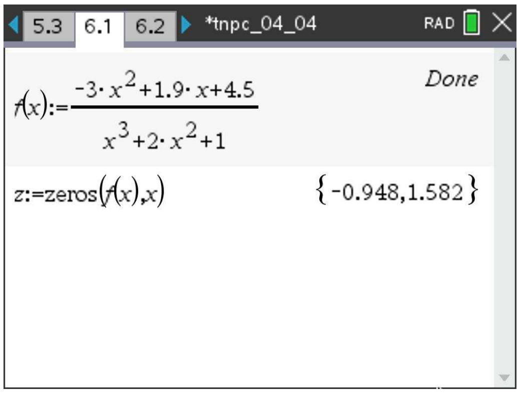
Consider the graph of g.

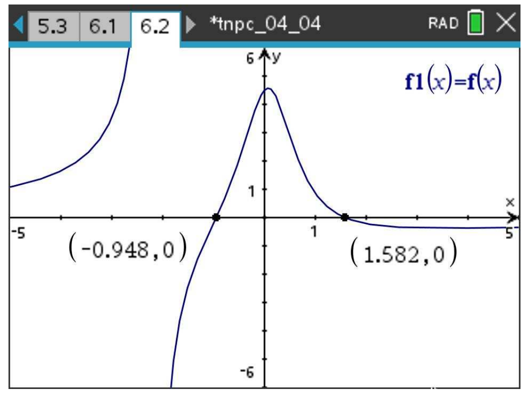
$$g(x) = -1 \quad \Rightarrow \quad x = -1, \ \ 0.5$$

(B) **(i)** Find all real zeros of f, or indicate there are no such values.

$$f(x) = \frac{-3x^2 + 1.9x + 4.5}{x^3 + 2x^2 + 1} = 0 \quad \Rightarrow \quad -3x^2 + 1.9x + 4.5 = 0$$

$x = -0.948, \quad 1.582$

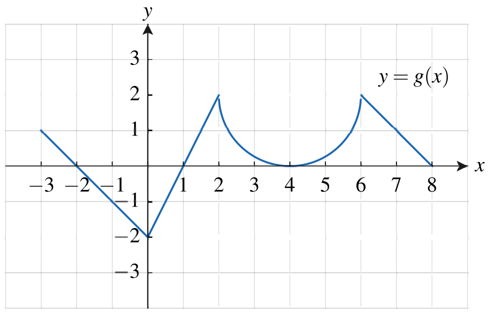




(ii) Determine the end behavior of f as x increases without bound. Express your answer using the mathematical notation of a limit.

$$\lim_{x \to \infty} f(x) = 0$$

(C)



(i) Determine if an inverse function of g can be constructed for all values of x in the closed interval $[2, 6]$.

No, an inverse function of g cannot be constructed for all values of x in the closed domain (of g) of $[2, 6]$.

(ii) Give a reason for your answer based on the definition of a function and the graph of g.

There is more than one value of x in the interval $[2, 6]$ that is mapped to the same output value.

8. The cost of an Uber ride in Boston is modeled by the function C given by

$$C(m) = \begin{cases} am + bm^2 & \text{if } 0 < m \leq 5 \\ d(m-5) + 25 & \text{if } m > 5 \end{cases}$$

where m is measured in miles and C is measured in dollars. Two Uber riders reported that for $m = 1$ the cost was \$9.00 and for $m = 3$, the cost was \$21.00.

(A) **(i)** Use the given data to write two equations that can be used to find the values for the constants a and b in the expression for $C(m)$.

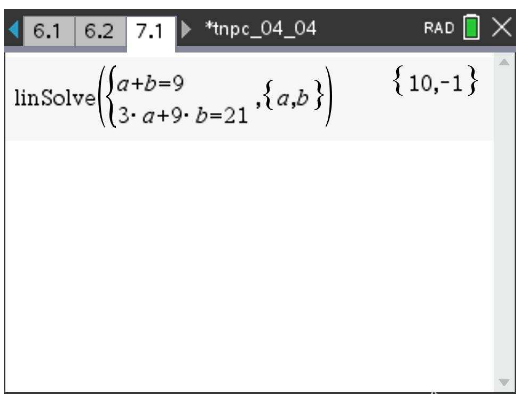
$$m = 1 \quad \Rightarrow \quad C(1) = a \cdot 1 + b \cdot 1^2 = a + b = 9$$

$$m = 3 \quad \Rightarrow \quad C(3) = a \cdot 3 + b \cdot 3^2 = 3a + 9b = 21$$

(ii) Find the values for a and b.

$$\begin{cases} a + b = 9 \\ 3a + 9b = 21 \end{cases} \Rightarrow \begin{cases} -3a - 3b = -27 \\ 3a + 9b = 21 \end{cases}$$

$$6b = -6 \quad \Rightarrow \quad b = -1 \qquad a - 1 = 9 \quad \Rightarrow \quad a = 10$$



(B) **(i)** Use the given data to find the average rate of change of the cost of a ride, in dollars per mile, from $m = 2$ to $m = 4$. Show the computations that lead to your answer.

$$\frac{C(4) - C(2)}{4 - 2} = \frac{24 - 16}{2} = \frac{8}{2} = 4 \text{ dollars/mile}$$

(ii) Interpret the meaning of your answer from (i) in the context of the problem.

On average, the cost of a ride increases 4 dollars per mile from $m = 2$ to $m = 4$.

(iii) The two pieces of the function C are connected at the transition point when $m = 5$. It is known that $\lim_{m \to 5} C(m) = 25$ and $C(6) = 27.5$. Consider the average rates of change of C from $m = 5$ to $m = p$ miles, where $p > 5$. Are these average rates of change less than or greater than the average rate of change from $m = 2$ to $m = 4$ miles found in (i)? Explain you reasoning.

The slope of the linear piece is: $27.5 - 25 = 2.5$

The average rates of change of C from $m = 5$ to $m = p$ miles are less than the average rate of change from $m = 2$ to $m = 4$ in part (i).

(C) Using the model C to predict the cost of an Uber ride, what is the maximum amount a rider could pay? Explain your reasoning.

For $m > 5$, the function C is linear and increasing.

Therefore, there is no maximum amount a rider could pay.