

# 3D Equations

## Teacher Notes and Answers

7 8 9 10 **11** 12



### Teacher Notes

This activity is designed to help students visualise and conceptualise the processes associated with solving equations of three variables. It is not intended to replace student's practice of the algorithmic approach, rather to introduce them to the process.

There are 5 equation sets stored in the TNS file. Teachers can edit the equations using the lists. Students select the equations from the set on the first page of the document (Page 1.1) and then navigate to page 1.2 to see a three dimensional graph of the equations. The slider on Page 1.2 can be used to successively show/hide pairs of equations.

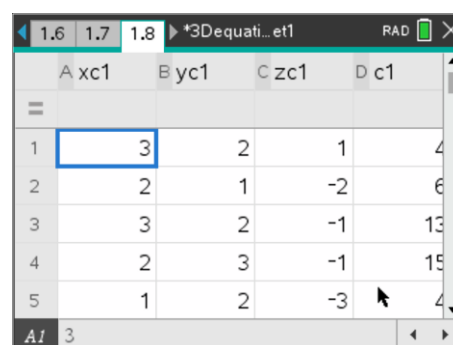
On page 1.3 students are presented with Equations 1 and 2 from the current equation set. They can use the slider to toggle between add and subtract and then the remaining sliders to adjust the multipliers to eliminate one of the variables using these two equations. The resulting equation is stored as Equation 4.

On page 1.4 students are presented with Equations 2 and 3 from the current equation set. Once again students can use the slider to toggle between add and subtract, then adjust the remaining sliders to change the multipliers and eliminate the same variable as the previous page. The resulting equation is stored as Equation 5.

If students eliminate a different variable on Page 1.3 compared with Page 1.4, they will soon see the problem when they get to page 1.5.

### Changing the equation sets:

The coefficients in each equation are stored in lists. For example the x coefficient in equation one is store in a list called: XC1. To edit all of the equations it is easiest to insert a Spreadsheet application and enter each of the lists names:



	A xc1	B yc1	C zc1	D c1
1	3	2	1	4
2	2	1	-2	6
3	3	2	-1	13
4	2	3	-1	15
5	1	2	-3	4

X coefficient of equation 1: XC1	Y coefficient of equation 1: YC1	Z coefficient of equation 1: ZC1	Constant (RHS of equation): C1
X coefficient of equation 2: XC2	Y coefficient of equation 2: YC2	Z coefficient of equation 2: ZC2	Constant (RHS of equation): C2
X coefficient of equation 3: XC3	Y coefficient of equation 3: YC3	Z coefficient of equation 3: ZC3	Constant (RHS of equation): C3

In the equation definitions, the coefficients are generated using: XC1[n], where n is the equation set number.

The definition of all the functions is located at the bottom of first page. The definitions have been written using a 'white' font colour so that they are not visible. Change the font colour to see how things have been defined and constructed.

If more equations are desired, change the maximum value of slider n (page 1.2) and simply add more equations to the bottom of each list in the spreadsheet.

## Introduction

An equation that describes the relationship between two variables such as  $x$  and  $y$  can be visualised on the Cartesian plane. Equations involving three variables such as  $x$ ,  $y$  and  $z$  can be described in three dimensions. Each of the three variable equations in this activity produces a plane, a flat surface. Solving simultaneous equations of three variables can be thought of as finding where all three flat surfaces intersect. This may sound complicated, it's not, it is a simple extension of solving simultaneous equations in two dimensions.

The TI-Nspire file 3D Equations contains sets of three equations. Your task is to find where the three planes intersect for each equation set.

## Instructions

Open the TI-nspire file: 3D Equations

On page 1.1, use the slider to select the first equation set.

$$\text{Equation 1: } 3x + 2y + z = 4$$

$$\text{Equation 2: } 2x - y - 3z = 11$$

$$\text{Equation 3: } x - 3y + 2z = 7$$

Make sure the equations in your document match those shown above.

Navigate to page 1.2. All three equations are displayed in three dimensions. There are several controls that help to visualise and manipulate the view of these three planes, explore them!

### Keyboard Commands:

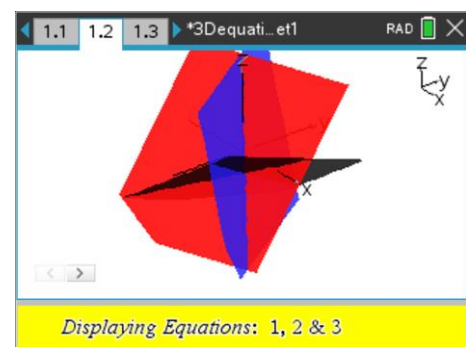
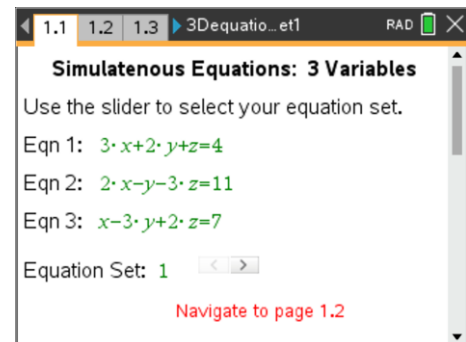
- $x$   $x$  axis oriented out of the page.
- $y$   $y$  axis oriented out of the page.
- $z$   $z$  axis oriented out of the page.
- $o$  default orientation (Letter 'o')
- $\times$  &  $\div$  zoom in and zoom out
- $a$  Auto rotation

Grab and move is also available to rotate graphs.

To grab, click in a white space then press CTRL followed by a centre click on the touchpad. Swipe your finger left / right / up / down on the touchpad. The rotation is slightly delayed depending on the resolution used to generate the 3D image. Press ESC to release the grab feature.

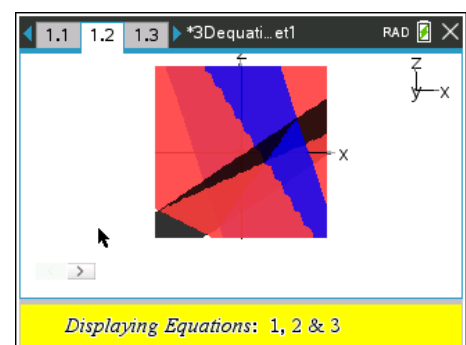
The 3D graph can provide an approximate indication of the point of intersection, the solution to the set of simultaneous equations.

Drag the axis around to get a sense of where the three planes intersect for Equation Set 1. The best viewing angle depends on the equation set. For this set of equations the 'y' view provides an indication for both the  $x$  and  $z$  values. (Shown opposite). The planes appear to intersect in what might be described as quadrant four in the  $x - z$  plane.



### Note:

Use the slider (bottom left) to toggle between different graph combinations. Displaying just two graphs at a time can help gain an understanding of how two planes intersect.



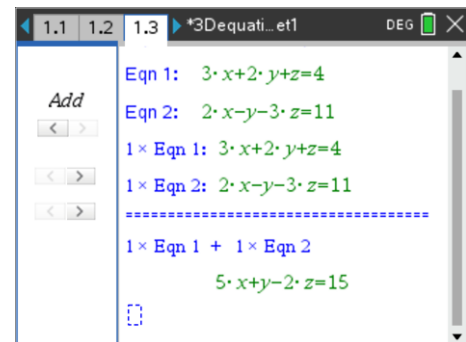
- Question: 1.** Explore the 3D representation of the Equation Set 1 and determine the nature of the point of intersection as follows. (Note: It is the positive end of each axis that is labelled.)  
 x coordinate: positive or negative? **Positive**  
 y coordinate: positive or negative? **Negative**  
 z coordinate: positive or negative? **Negative**

Navigate to page 1.3. The aim here is to remove one of the variables. For this page focus on removing 'y' using the two equations. (1 & 2)

**Note:** If the equations are not displayed, press Ctrl + A (select all) then choose Evaluate from the Actions menu.

Decide whether equation 1 and 2 need to be added or subtracted. The top slider selects Add / Subtract.

The next task is to determine how much to multiply equation 1, 2 or both by in order to eliminate y. If you're not sure, try clicking a couple of times, the equations update so too the combination of the equations at the base of the screen. When the equation at the base of the screen doesn't include 'y', then you have achieved your goal. This new equation will be called: **Equation 4**.



- Question: 2.** Write down your equation 4.

**Answer:**  $7x - 5z = 26$  Students may have multiples of this answer:  $14x - 10z = 52$  etc.

- Question: 3.** Transpose your equation to make z the subject.

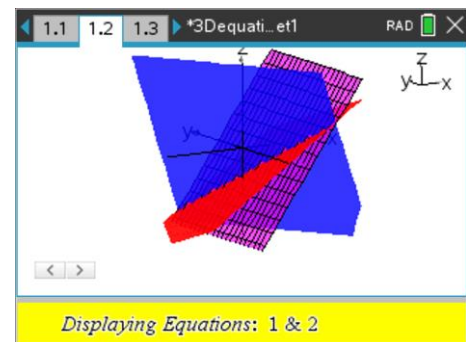
**Answer:**  $z = \frac{7x - 26}{5}$  Note that this equation is required to enter and view the graph.

Navigate to page 1.2. Use the slider to display equations 1 and 2 only.

Use the TAB key to navigate to the z entry line and type in your new equation. (Question 3) The new equation will appear as a 'wire lattice' to make it easy to identify. (See opposite).

Experiment with the view to see how your new equation interacts and intersects with equations 1 and 2.

Once you have finished exploring, switch **your** equation off or delete it from the z entry line.



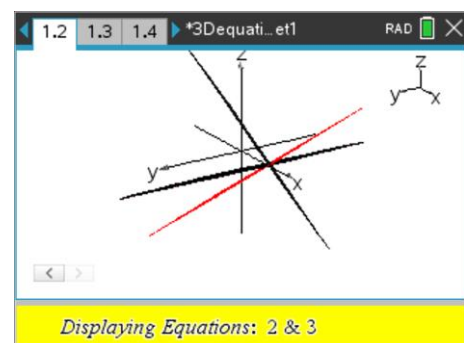
- Question: 4.** Explain how and where your new equation intersects with equations 1 and 2.

Students should see that the planes intersect along a single line. Some views are better than others.

#### Teacher Notes:

It is particularly valuable to have students change the 3D view so that they can see the line along which the two graphs intersect. An example is shown here for Graphs 2 and 3 with the corresponding equation 4.

It is clear to see that all three planes intersect along a single line or axis. If students imagine this line as an axis in 3D space, then a second line or



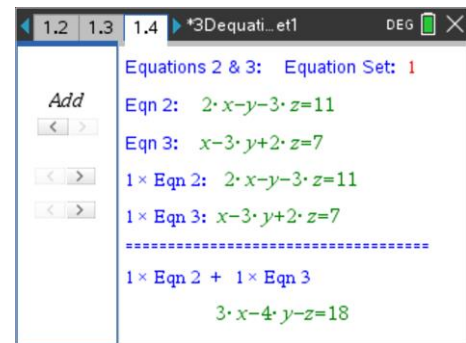
axis created by the intersection of the two planes corresponding to equations 1 and 2, they should then be able to imagine a point where these two axes meet.

This process will help students develop a conceptual understanding of equations in 3D.

Navigate to page 1.4. The aim here is to remove one of the variables. The variable to be removed **must** be the same as for the previous page. For this page focus on removing 'y' using the two equations. (2 & 3)

Decide whether equation 2 and 3 need to be added or subtracted. The top slider selects Add / Subtract.

The next task is to determine how much to multiply equation 2, 3 or both in order to eliminate y. This new equation will be called: **Equation 5**.



**Question: 5.** Write down your equation 5.

**Answer:**  $5x - 11z = 26$  As per question 3, students may have a multiple of this equation.

**Question: 6.** Transpose the equation to make z the subject.

**Answer:**  $z = \frac{5x - 26}{11}$

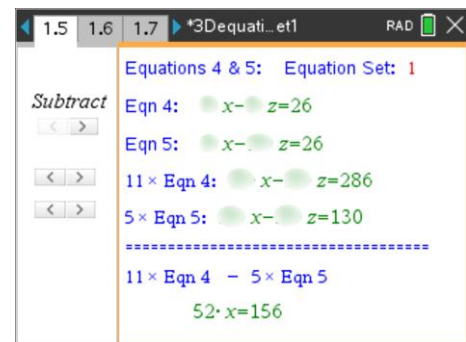
**Question: 7.** Graph your new equation on Page 1.2 whilst displaying equations 2 and 3. What do you notice?

As before, students should see that the planes intersect along a single line. This now leads to the notion that these two lines will intersect at a point that corresponds to the solution to the simultaneous equations.

Navigate to page 1.5. If you have successfully completed pages 1.3 and 1.4, eliminating the same variable from each pair to create equations 4 and 5, you should see two equations here with two variables!

Your aim here is to remove one of the variables leaving a single equation with just one variable, as shown opposite.

The coefficients on each equation have been blurred on purpose!



**Question: 8.** Write down your final equation (one variable) and determine the corresponding value for this variable.

**Answer:**  $52x = 156$ , therefore  $x = 3$

**Question: 9.** Use equation 4 and your answer to question 8 to determine the corresponding second variable.

**Answer:**  $7(3) - 5z = 26$   
 $z = -1$

**Question: 10.** Return to page 1.1 and use equation 1 to determine the final variable.

**Answer:**  $3(3) + 2y + (-1) = 4$  therefore  $y = -2$ .

**Question: 11.** Write down the coordinates  $(x, y, z)$  for the point where the three planes intersect. Compare the values with your answer to question 1.

**Answer:**  $(x, y, z) = (3, -2, -1)$  This correlates with the visual inspection  $(+, -, -)$

**Question: 12.** Verify your results by substituting the values into equations 2 and 3.

**Answer:** Equation 2:  $2(3) - (-2) - 3(-1) = 11$  True

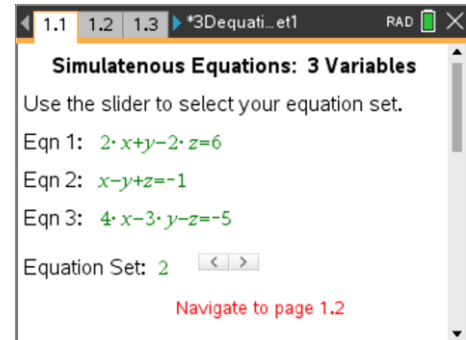
Equation 3:  $(3) - 3(-2) + 2(-1) = 7$  True

Navigate to page 1.1. Use the slider to generate equation set 2.

Navigate to page 1.2 and estimate the nature of  $x, y$  and  $z$ .  
(positive or negative)

Use the environments on pages 1.3, 1.4 and 1.5 to determine the where these three equations intersect.

Remember you can check equations 4 and 5 using the graph on page 1.2



**Question: 13.** Determine solutions to each of the four remaining equation sets. Try eliminating the variables in a different order; remember however, the same variable must be removed from equation sets on pages 1.3 and 1.4.

**Answers:**

- Equation Set 2:  $(x, y, z) = (2, 4, 1)$  From 3D graph:  $(+, +, +)$
- Equation Set 3:  $(x, y, z) = (2, 3, -1)$  From 3D graph:  $(+, +, -)$
- Equation Set 4:  $(x, y, z) = (2, 4, 1)$  From 3D graph:  $(+, +, +)$
- Equation Set 5:  $(x, y, z) = (4, -3, -2)$  From 3D graph:  $(+, -, -)$