

Paper Folding

Answers

7 8 9 10 11 12



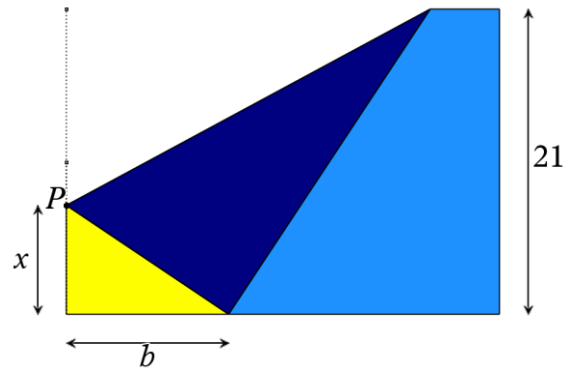
Introduction

An A4 piece of paper measures 29.7cm x 21.0cm. The page is oriented in 'landscape' format. The top left corner of the page is folded such that the corner just touches the base of the page. A triangle is formed in the bottom left corner. The height of the triangle is denoted by x and the base b .

The aim of this investigation is to determine the maximum area that can be formed with this triangle.

Teacher Notes:

Students can benefit enormously by starting this activity with pencil and paper. Students can fold the paper and determine the area of the triangle for their specific fold. The actual folding and un-folding of the triangle helps students realise the relationship between x and the length of the hypotenuse.



Forming an Equation

Question: 1.

Determine an expression for the hypotenuse of the triangle in terms of x .

$$\text{hypotenuse} = 21 - x$$

Question: 2.

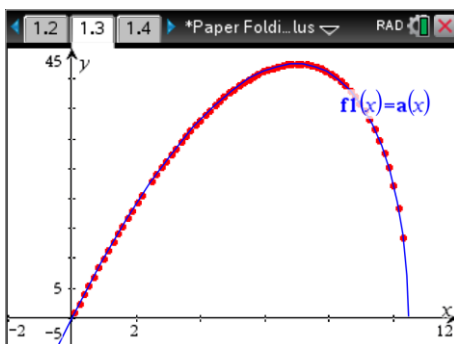
Determine an expression for the base of the triangle in terms of x and state any domain restrictions.

$$b = \sqrt{21(21-2x)} \quad \text{Domain Restriction: } x < \frac{21}{2} \quad (\text{Half the height of the page})$$

Question: 3.

Define a function $a(x)$ for the area of the triangle in terms of x .

$$a(x) = \frac{x}{2} \sqrt{21(21-2x)}$$



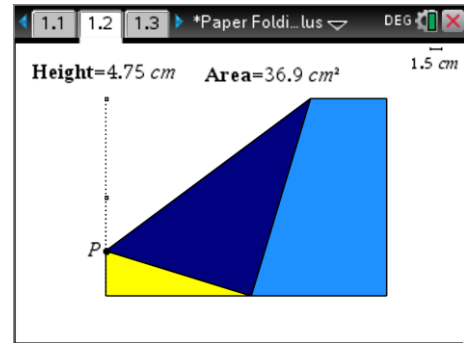
Validating the Equation

Open the TI-Nspire file “Paper Folding”.

Navigate to page 1.2

Grab point P and move it up and down. As point P moves up and down the height and area of the triangle is being collected automatically.

Navigate to page 1.3 and graph the function $a(x)$ and confirm that it passes through the data points generated.



Question: 4.

Determine the derivative of the function: $a(x)$.

$$\begin{aligned} a'(x) &= \frac{21\sqrt{21} - 3\sqrt{21}x}{2\sqrt{21} - 2x} \\ &= \frac{3\sqrt{21}(7 - x)}{2\sqrt{21} - 2x} \end{aligned}$$

It is assumed that students will use CAS to determine the derivative.

Question: 5.

Determine the value for x for which the area is a maximum.

$$\begin{aligned} \frac{3\sqrt{21}(7 - x)}{2\sqrt{21} - 2x} &= 0 \\ x &= 7 \end{aligned}$$

In the factorised form students should see that $x = 7$ is the solution, without using solve.

Question: 6.

Determine the maximum area of the triangle.

$$a(7) = \frac{49\sqrt{3}}{2}$$

Question: 7.

Suppose the ‘height’ of the paper is changed from 21cm to h cm. Determine the value of x for which the area of the triangle is a maximum and the corresponding area.

$$x = \frac{h}{3}$$

Teacher Notes:

The solution is quite elegant. Students should also notice that this also produces angles of $\frac{\pi}{6}$ and $\frac{\pi}{3}$.