Binomial Probability In Baseball

Name _____

Class _____

Exploring the Problem

During regular season, the Boston Red Sox and Cleveland Indians played against each other 7 times. The Red Sox won 6 of these games.

Expectations are developed based on previous performance. Overlooking the other variables affecting baseball, the wins/losses information may be used to develop probabilities associated with winning and losing.

1. Using the wins/losses information above, determine the following:

a) $p = P(\text{Red Sox win}) =$ _____

b) $q = P(\text{Red Sox lose}) =$ _____

2. For a single probability event with only two outcomes, $p + q = 1$.

☐ Always ☐ Sometimes ☐ Never

Developing the Pattern

Binomial events have only two outcomes. So, binomial experiments must meet the following conditions:

- Each trial has exactly 2 outcomes, *p* and *q* (win/lose, pass/fail, true/false, on/off).
- The number of trials is fixed (*n*).
- The outcome of each trial is independent (example: In a coin toss, what turns up on the 2nd toss is not affected by the 1st toss.).
- The values of *p* and *q* do not change.

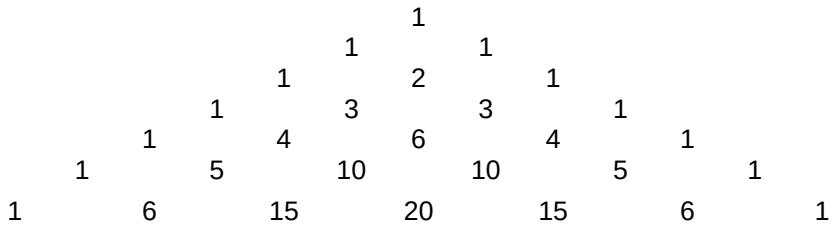
If the conditions of a binomial experiment are met, the variable *n* represents the number of trials and the variable *x* represents the number of successes. The binomial expansion of $(p + q)^n$ can be used to represent these situations.

Example:

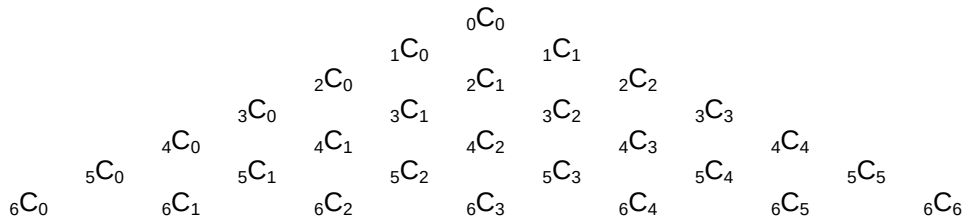
A student takes a multiple choice quiz with 5 questions. Each question has 4 choices. She hasn't studied and will guess on every question. In order to pass this quiz, she must get 4 of the questions correct.

Given that she is guessing, assume $p = \frac{1}{4}$ and $q = \frac{3}{4}$. Also, $n = 5$ because there are 5 questions.

Pascal's Triangle



Below is another form of Pascal's Triangle, where each row now represents the number of trials in a binomial probability experiment, is given below. The variable x decreases from n to 0 as you move left to right across a row.



For the student's quiz, $n = 5$, and she needs to answer 4 questions correctly, so $x = 4$.

Answer the following questions based on binomial a:

3. Find the binomial expansion of $(p + q)^5$ using Pascal's Triangle:

4. Substitute $p = \frac{1}{4}$ and $q = \frac{3}{4}$ into the 4th term obtained by counting down terms 5th, 4th, 3rd, 2nd, 1st, and 0th, reading from left to right. Evaluate this term after making the substitution.

5. Find the probability that the student answers 4 of 5 quiz questions correctly using $_nC_x\, p^x\, q^{(n-x)}$. To enter $_nC_x$ on the calculator, type the number for n, press MATH and under **PRB** select the **nCr** command, then type the number for x.

6. Find the probability that the student answers 4 of 5 quiz questions correctly using **binomPdf(n,p,x)**. To use this command, press 2nd [DISTR] and select **binompdf(**. Enter the numbers for *n*, *p*, and *x* separated by commas.

7. Did your results for the two calculations for the student's quiz match?

Extending the Pattern

Now, answer the following questions about the Red Sox in the American League Championship, where the first team to win 4 games is the champion.

8. Find *P*(Red Sox win 4 of 4 games).

9. Would you have expected that seven games were played with the Red Sox winning their fourth game with game 7? Explain.

10. Find *P*(Red Sox win 4 of 7 games).

11. Is finding the probability of winning 4 of 7 games using straight forward binomial probability as performed in this activity a good model for the 7-game situation? How does the 4 of 7 games situation differ from the 4 of 4 situation?

12. How many games would you have expected to be played for a champion to be determined for the American League?

13. Identify at least 3 variables in baseball or any sport that make using past performance for determining probability problematic.