Science Objectives

- Students will determine the mechanical advantages and efficiencies of simple machines.
- Students will relate the factors that affect the mechanical advantage of an inclined plane and a lever.
- Students will develop an understanding of the mechanical advantage of an inclined plane and a lever as they relate to the concepts of force, torque, and static equilibrium.

Vocabulary

- first-class lever
- inclined plane
- static equilibrium
- force
- mechanical advantage
- torque

About the Lesson

- This lesson has students explore the relationship between resistance force and effort force for inclined planes and levers.
- Students first gather real-world data for the effort force required to hold a mass stationary on an inclined plane versus the height and length of the plane. Then students use simulation to explore the mechanical advantage of first-class levers.
- Students predict and test formulas for the mechanical advantage of inclined planes and levers, and then apply their formulas in real-world applications.

TI-Nspire™ Navigator™

- Send out the *Mechanical_Advantage.tns* file.
- Monitor student progress using Screen Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- *Mechanical_Advantage.tns* document
- TI-Nspire™ Technology
- Vernier Dual-Range Force Sensor
- EasyLink™ or Go!™Link interface
- small metal cart with wheels (or other mass that slides easily)
- meter stick or tape measure
- piece of wood and several books for making an inclined plane
- string
- pen or pencil

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Collect data with probes

Tech Tip:

Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Mechanical_Advantage_Student.doc
- Mechanical_Advantage_Student.pdf

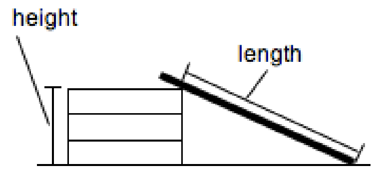
TI-Nspire document

- Mechanical_Advantage.tns

Discussion Points and Possible Answers

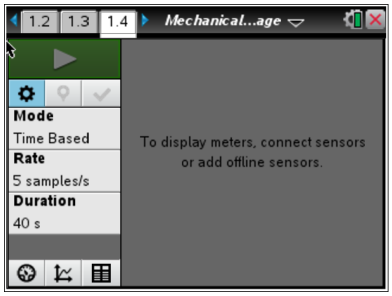
Problem 1: Mechanical advantage of inclined plane and lever

Move to pages 1.3–1.6.



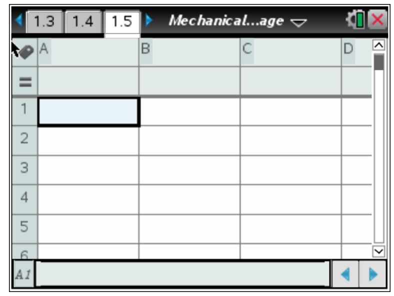
1. Discuss how students will change the height of the inclined plane by varying the number of books supporting the wood.
2. They should use a wide variety of angles, including several that are quite steep and several that are quite shallow.

Move to page 1.4.



3. Next, students should gather the data using the blank DataQuest application on page 1.4. They should connect the force sensor to the EasyLink or Go!Link interface, and then connect the interface to their handheld or computer.
4. Then students should zero the force sensor (**Menu > Experiment > Set Up Sensors > Zero**). Then, they should use the string to connect the cart to the hook on the bottom of the force sensor. (Make sure the switch on the force sensor is set to the correct range. The range students use will depend on the mass they use.) They should let the cart hang from the force sensor without touching anything. Once the weight reading on the force sensor has stabilized, students should record the cart's weight on a piece of scrap paper.
5. Next, students should then set up the sensor to **Events with Entry** mode (**Menu > Experiment > Collection Mode > Events with Entry**) and begin the experiment by selecting Start Data Collection. (Note: The data shown here are simulated; students' data will vary.)
6. Next, students should place the cart on the inclined plane, wait for the force reading to stabilize, and record a data point. They should measure the height of the ramp and enter that value as the "event" value.

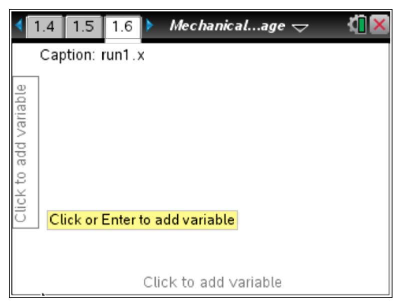
Move to page 1.5.



7. Students should repeat Step 6 at least five times, changing the height of the ramp at each repetition. (Make sure students measure the height of the ramp at each step and use that value for the "event" value.) Once data collection is complete, students can stop the data collection and disconnect the force sensor. If students' data do not automatically appear in the *Lists & Spreadsheet* application on page 1.5, they should assign Column A to the height data (stored in the variable **run1.event**) and Column B to the force data (stored in **run1.force**). To assign data to a column, students should select the formula bar (light gray square) of the column and press [var]. They should then select the variable they want to assign to the column.

8. Define the variables **hratio** (Column C) and **wratio** (Column D). The variable **hratio** should be the ratio of the height of the inclined plane at each data point to the length of the ramp (which students measured in Step 2). The variable wratio should be the ratio of each measured force value to the weight of the cart.

Move to page 1.6.



9. Page 1.6 contains an empty *Data & Statistics* application. Students should make a plot of **hratio** vs. **wratio**.

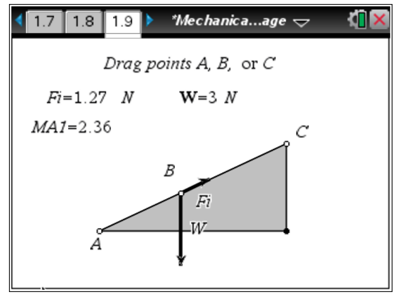
Move to page 1.7.

Have students answer the question on either the handheld, on the activity sheet, or both.

Q1. What does the shape of the graph of **hratio** vs. **wratio** imply about the relationship between the height of an inclined plane and the amount of force required to move an object up the plane?

Answer: The data should fall along a straight line. This implies that, as the inclined plane's height increases, the amount of force required to move the object up the plane (as a fraction of the object's weight) increases proportionally.

Move to pages 1.8 and 1.9.



10. Page 1.8 introduces students to the inclined plane simulation on page 1.9. In the simulation, the weight of the box, **B**, is 3 N. Students can drag point **B** up the incline, drag point A to change the slope, or drag point **C** to change the height of the plane. The effort force (**W**), resistance force (*Fi*), and mechanical advantage (*MA1*) are given in the simulation. Students should vary the height and slope of the inclined plane and observe the effects on the mechanical advantage.

Move to pages 1.10 – 1.12.

Have students answer the questions on either the handheld, on the activity sheet, or both.

Q2. How can you increase the mechanical advantage of the inclined plane?

<u>Answer</u>: by increasing the length of the sloped side of the inclined plane or by decreasing the height of the inclined plane

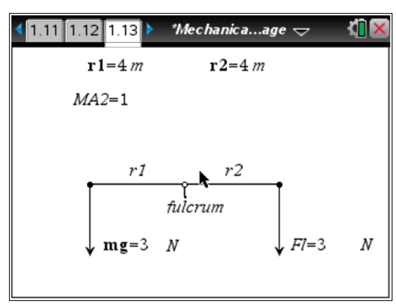
Q3. What is the formula for calculating the mechanical advantage of this inclined plane?

<u>Answer</u>: The mechanical advantage of a machine is defined as the ratio of the resistance force to the effort force. For this inclined plane, the resistance force is *Fi*, and the effort force is ***W***. Therefore, the mechanical advantage of this inclined plane is equal to $\frac{\boldsymbol{W}}{\text{Fi}}$.

Q4. Verify the formula you figured out in Question 3 using the simulation on page 1.8.

<u>Answer</u>: Students should use the **Text** tool (**Menu > Actions > Text**) to enter $\frac{W}{Fi}$ somewhere on page 1.8. Then, they should use the **Calculate** tool (**Menu > Actions > Calculate**) to determine the value of this ratio. Finally, they should compare the value they obtain with the value given for *MA1* to confirm that their derived formula is correct.

Move to pages 1.12 and 1.13.



11. Page 1.13 shows a first-class lever. Change the position of the fulcrum and observe the effect of the fulcrum position on the effort force, *Fl*. The weight of the box is **mg** = 3N.
    Students should vary the position of the fulcrum and observe the effects on the mechanical advantage (*MA2*).

Move to pages 1.14 and 1.15.

Have students answer the questions on either the handheld, on the activity sheet, or both.

Q5. How can you increase the mechanical advantage of the lever?

Answer: by moving the fulcrum as close to the box as possible, thus increasing effort arm **r2** and decreasing resistance arm **r1**

Q6. What is the formula for calculating the mechanical advantage of this lever?

Answer: The mechanical advantage of a machine is defined as the ratio of the resistance force to the effort force. For this lever, the resistance force is *Fl*, and the effort force is **mg**. Therefore, the mechanical advantage of this inclined plane is equal to $\frac{\boldsymbol{mg}}{\mathrm{Fl}}$. Encourage student discussion of how they identified the effort and resistance forces.

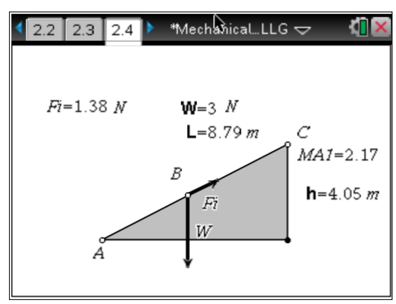
Q7. Verify the formula you figured out in Question 6 using the simulation on page 1.13.

Answer: Students should use the **Text** tool to enter $\frac{mg}{Fl}$ somewhere on page 1.13. Then, they should use the **Calculate** tool to determine the value of this ratio. Finally, they should compare the value they obtain with the value given for *MA2* to confirm that their derived formula is correct.

Problem 2: Another formula for the mechanical advantage of an inclined plane

Move to pages 2.1 – 2.5.

12. Read the text on 2.1 then use the simulation on page 2.4 to answer questions 8 to 10. Write your answers here or in the .tns file. Students can use the blank calculator application on page 2.6 to answer Question 10.



Q9. Test your derived formula from Question 8 using the simulation on page 2.4.

Answer: On page 2.4, students should use the **Text** and **Calculate** tools to verify that the ratio of **L** to **h** is equal to the ratio of **W** to *Fi*.

Q10. A dump truck weighing 15,000 N is at the bottom of a mountain. The height of the mountain is 100 m, and the sloped road the truck is going to drive up is 3,000 m long. What is the mechanical advantage of the road? How much effort force is needed to push the dump truck up the mountain?

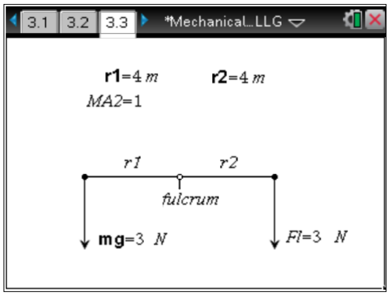
Answer: Students should use the Calculator application on page 2.6 to answer this question. To find mechanical advantage, use the formula derived in question 8, as shown here: $MA = \frac{\mathbf{L}}{\mathbf{h}} = \frac{3{,}000\ m}{100\ m} = 30$. To calculate the effort force, rearrange the equation for mechanical advantage, as shown here: $Fi = \frac{\mathbf{W}}{\mathbf{MA}} = \frac{15{,}000\ N}{30} = 500\,N$.

Problem 3: Another formula for the mechanical advantage of a first-class lever

Move to pages 3.1 and 3.4.

Have students answer the questions on either the handheld, on the activity sheet, or both.

13. Use the simulation on page 3.3 to answer questions 11-13. Students can use the blank calculator application on page 3.5 to answer Question 13.



Q11. Derive a formula for the mechanical advantage of a first-class lever in terms of the lengths of the two arms of the lever, **r1** and **r2**. (Hint: The torques created by both forces, effort and resistance, are equal.)

Answer: The torque produced by the resistance force is **mg · r1**. The torque produced by the effort force is *Fl* **· r2**. Setting these expressions equal to each other yields **mg · r1** = *Fl* **· r2**. Rearranging yields the following: $MA = \frac{\mathbf{mg}}{Fl} = \frac{\mathbf{r2}}{\mathbf{r1}}$. Thus, for a first-class lever, the mechanical advantage is equal to the ratio of the effort arm length to the resistance arm length. If you wish, you may have students first examine the simulation and infer the relationship between *MA*, **r1,** and **r2**. They should be able to recognize that *MA* is equal to the ratio of **r2** to **r1**. Once they have identified this relationship, you can have them work "backward" to identify the equality of the two torque equations.

Q12. Test your derived formula from Question 11 using the simulation on page 3.3.

Answer: On page 3.3, students should use the **Text** and **Calculate** tools to verify that the ratio of **r2** to **r1** is equal to the ratio of **mg** to *Fl*.

Q13. A student uses a lever to lift a 20 kg box. The distance between the fulcrum and the point at which the student grabs the lever is 0.5 m. If the student applies a force of 50 N to the lever, how far is the box from the fulcrum? What is the mechanical advantage of this lever?

Answer: Students should use the **Calculator** application on page 3.5 to answer this question. To find the distance, students should use the fact that torques are equal.
Thus, *Fl* · h = m · g · d, where *Fl* = 50 N, m = 20 kg, and d = 0.5 m. (In this equation, **h** is the distance between the fulcrum and the effort, and d is the distance between the fulcrum and the load.)

Solving for d yields the following: $d = \frac{Fl \cdot h}{m \cdot g} = \frac{(50\ N)(0.5\ m)}{(20\ kg)(9.8\ m\ /\ s^2)} = 0.13\ m$.

Therefore, according to the equation derived above, the mechanical advantage of the lever is as follows: $MA = \frac{h}{d} = \frac{0.5}{0.13} = 3.85$.

TI-Nspire Navigator Opportunities

Use TI-Nspire Navigator to capture screen shots of student progress and to retrieve the file from each student at the end of the class period. The student questions can be electronically graded and added to the student portfolio.

Wrap Up

Make sure the concept of balancing torques is firm in their understanding and not balancing forces.

Assessment

- Formative assessment will consist of questions from the student handout.
- Summative assessment will consist of questions/problems on the chapter test.