

String Graphs – Part 3

Teacher Notes and Answers

7 8 9 **10** 11 12



TI-Nspire



Investigation



Student



45 min

Aims

- Locate the points of intersection between consecutive lines.
- Determine a rule connecting consecutive points of intersection.

Simultaneous Equations

In this activity you will locate the points of intersection between consecutive lines in the Strings 2 activity.

The first two equations from this activity passed through the points:

Line 1: (-10, 10) & (1, 1) Equation 1: $y = -\frac{9}{11}x + \frac{20}{11}$

Line 2: (-9, 9) & (2, 2) Equation 2: $y = -\frac{7}{11}x + \frac{36}{11}$

The first step eliminates y from the two equations. The point where the two lines intersect, by definition, will share the same values for both x and y at the point (x, y) . As both equations are equal to y , then it follows that:

$$-\frac{9}{11}x + \frac{20}{11} = -\frac{7}{11}x + \frac{36}{11} \quad (\text{Equation 1} = \text{Equation 2})$$

As everything on the left hand side (LHS) of the equation is divided by 11, so too everything on the right hand side (RHS), then it follows that the numerators must be equal:

$$-9x + 20 = -7x + 36$$

Question: 1.

Solve the linear equation $-9x + 20 = -7x + 36$ for x and hence determine the point of intersection between the first and second graph.

By hand: $-9x + 20 = -7x + 36$ (+9x) Point of intersection: (-8, 92/11)
 $20 = 2x + 36$ (-36)
 $-16 = 2x$ (÷2)
 $x = -8$

Question: 2.

The second and third equations from String Graphs 2 were: $y = -\frac{7}{11}x + \frac{36}{11}$ and $y = -\frac{5}{11}x + \frac{48}{11}$.

Determine the coordinates of the point where these two graphs intersect.

Point of intersection (-7, 83/11)

Question: 3.

The third and fourth equations from String Graphs 2 were: $y = -\frac{5}{11}x + \frac{48}{11}$ and $y = -\frac{3}{11}x + \frac{56}{11}$.

Determine the coordinates of the point where these two graphs intersect.

Point of intersection (-4, 68/11)

Determining Equations

Open your String Graphs 2 document.¹

Navigate to Page 2.2. [Calculator Application]

The first two equations can be recalled by typing $f(x,1)$ and $f(x,2)$.

To see the elimination of y , type in: $f(x,1) = f(x,2)$.

A common denominator can be obtained by:

Menu > Number > Fraction tools > Common Denominator

Once the equation has been expressed with a common denominator both sides of the equation can be multiplied by this denominator. To do this simply press the multiplication key and the calculator will automatically reference the previous answer.

x 11 (See opposite)

In this case “Ans” represents the previous equation.

Both sides of the equation will now be surrounded with parenthesis. Parenthesis can be removed by ‘expanding’.

Menu > Algebra > Expand

Select the previous result and copy it into the expand() command.

The resulting equation is now the same as the one solved in Question 1. It is possible to continue solving this equation on the calculator using the same process.

For example the next step could be “add $9x$ ” to both sides.

To achieve this simply type: $+9x$

Continue this process to determine the value of x .

Strings 2 DEG

$$f(x,2) = \frac{36}{11} - \frac{7 \cdot x}{11}$$

$$f(x,1) = f(x,2) \quad \frac{20}{11} - \frac{9 \cdot x}{11} = \frac{36}{11} - \frac{7 \cdot x}{11}$$

$$\text{comDenom}\left(\frac{20}{11} - \frac{9 \cdot x}{11} = \frac{36}{11} - \frac{7 \cdot x}{11}\right)$$

*Strings 2 DEG

$$f(x,1) = f(x,2) \quad \frac{20}{11} - \frac{9 \cdot x}{11} = \frac{36}{11} - \frac{7 \cdot x}{11}$$

$$\text{comDenom}\left(\frac{20}{11} - \frac{9 \cdot x}{11} = \frac{36}{11} - \frac{7 \cdot x}{11}\right)$$

$$\frac{20 - 9 \cdot x}{11} = \frac{36 - 7 \cdot x}{11}$$

Ans: 11

*Strings 2 DEG

$$\text{comDenom}\left(\frac{20}{11} - \frac{9 \cdot x}{11} = \frac{36}{11} - \frac{7 \cdot x}{11}\right)$$

$$\frac{20 - 9 \cdot x}{11} = \frac{36 - 7 \cdot x}{11}$$

$$\left(\frac{20 - 9 \cdot x}{11} = \frac{36 - 7 \cdot x}{11}\right) \cdot 11$$

$$-(9 \cdot x - 20) = -(7 \cdot x - 36)$$

$$\text{expand}(-(9 \cdot x - 20) = -(7 \cdot x - 36))$$

*Strings 2 DEG

$$\frac{20 - 9 \cdot x}{11} = \frac{36 - 7 \cdot x}{11}$$

$$\left(\frac{20 - 9 \cdot x}{11} = \frac{36 - 7 \cdot x}{11}\right) \cdot 11$$

$$-(9 \cdot x - 20) = -(7 \cdot x - 36)$$

$$\text{expand}(-(9 \cdot x - 20) = -(7 \cdot x - 36))$$

$$20 - 9 \cdot x = 36 - 7 \cdot x$$

Ans+9x|

¹ String Graphs 2 TI-Nspire file:

If you did not save this file, the main thing you need is the equations, including the general equation for the straight line graphs.

An efficient way to solve simultaneous equations on the calculator is to use the “Solve” command.

Menu > Algebra > Solve

One of the equations pairs we are trying to solve is the for the point of intersection between the first two lines:

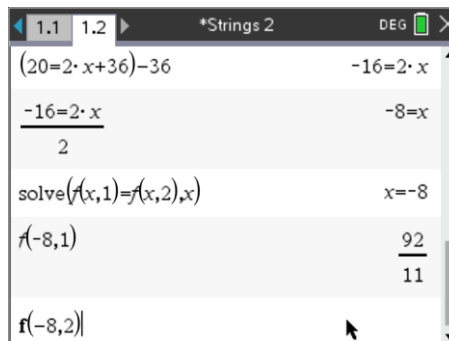
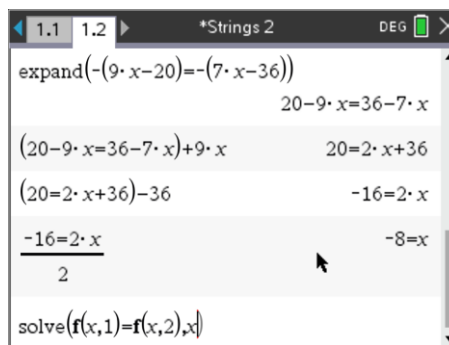
$$f(x,1) \text{ and } f(x,2) .$$

The solve command does all the calculations and arrives immediately at the answer: $x = -8$.

This is only the x coordinate (abscissa), the y value (ordinate) can be obtained by substituting -8 in for x .

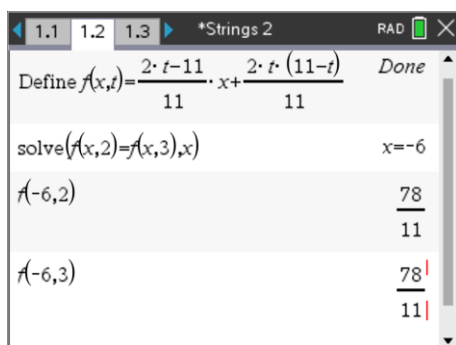
The screen shown opposite shows this as $f(-8,1)$ and the corresponding result: $\frac{92}{11}$.

The second line should pass through the same point. See what happens when you substitute -8 in for x in $f(x,2)$.



Question: 4.

Use the solve command to determine the point of intersection for lines 2 and 3: $f(x,2) = f(x,3)$.



Note that $f(-6,3)$ confirms the point of intersection.

Question: 5.

Complete the table below. (The first couple of equations have been entered for you)

Eqn. No.	Point 1	Point 2	Equation	Eqn. Pair (t)	x-coordinate Point of Intersection	y-coordinate Point of intersection	Point of Intersection
1	(-10, 10)	(1, 1)	$y = -\frac{9}{11}x + \frac{20}{11}$	1	$x = -8$	$y = \frac{92}{11}$	$\left(-8, \frac{92}{11}\right)$
2	(-9, 9)	(2, 2)	$y = -\frac{7}{11}x + \frac{36}{11}$				
2	(-9, 9)	(2, 2)	$y = -\frac{7}{11}x + \frac{36}{11}$	2	$x = -6$	$y = \frac{78}{11}$	$\left(-6, \frac{78}{11}\right)$
3	(-8, 8)	(3, 3)	$y = -\frac{5}{11}x + \frac{48}{11}$				
3	(-8, 8)	(3, 3)	$y = -\frac{5}{11}x + \frac{48}{11}$	3	$x = -4$	$y = \frac{68}{11}$	$\left(-4, \frac{68}{11}\right)$
4	(-7, 7)	(4, 4)	$y = -\frac{3}{11}x + \frac{56}{11}$				
4	(-7, 7)	(4, 4)	$y = -\frac{3}{11}x + \frac{56}{11}$	4	$x = -2$	$y = \frac{62}{11}$	$\left(-2, \frac{62}{11}\right)$
5	(-6, 6)	(5, 5)	$y = -\frac{1}{11}x + \frac{60}{11}$				
5	(-6, 6)	(5, 5)	$y = -\frac{1}{11}x + \frac{60}{11}$	5	$x = 0$	$y = \frac{60}{11}$	$\left(0, \frac{60}{11}\right)$
6	(-5, 5)	(6, 6)	$y = \frac{1}{11}x + \frac{60}{11}$				
6	(-5, 5)	(6, 6)	$y = \frac{1}{11}x + \frac{60}{11}$	6	$x = 2$	$y = \frac{62}{11}$	$\left(2, \frac{62}{11}\right)$
7	(-4, 4)	(7, 7)	$y = \frac{3}{11}x + \frac{56}{11}$				
7	(-4, 4)	(7, 7)	$y = \frac{3}{11}x + \frac{56}{11}$	7	$x = 4$	$y = \frac{68}{11}$	$\left(4, \frac{68}{11}\right)$
8	(-3, 3)	(8, 8)	$y = \frac{5}{11}x + \frac{48}{11}$				
8	(-3, 3)	(8, 8)	$y = \frac{5}{11}x + \frac{48}{11}$	8	$x = 6$	$y = \frac{78}{11}$	$\left(6, \frac{78}{11}\right)$
9	(-2, 2)	(9, 9)	$y = \frac{7}{11}x + \frac{36}{11}$				
9	(-2, 2)	(9, 9)	$y = \frac{7}{11}x + \frac{36}{11}$	9	$x = 8$	$y = \frac{92}{11}$	$\left(-8, \frac{92}{11}\right)$
10	(-1, 1)	(10, 10)	$y = \frac{9}{11}x + \frac{20}{11}$				



As you answer more and more questions in this calculator document you will need to start saving your work. Press **DOC > File > Save As** and save your document as "String Graphs 2".

Question: 6.

Write a relationship between the equation pair (t) and the x coordinate of the point of intersection between consecutive curves. Write your rule in the form $x =$

Answer: $x = 2t - 10$

Question: 7.

It is possible to check your answer to the previous question using the 'solve' command.

Use: $\text{solve}(f(x,t) = f(x,t+1), x)$ to get the rule. Compare the answer to the one you obtained in Question 6.

Answer: $x = 2t - 10$ The answer is the same as Question 6. The solve command is comparing consecutive equations by using t and $t + 1$.

The relationship between the equation pair (t) and the y coordinate is not linear; this makes it a bit harder to work out the rule. However, as we have the rule for the x coordinate, we can determine the rule for the y coordinate, once again, using the calculator. Earlier in this activity we substituted in numbers for x . You can also substitute an entire rule for x .

Question: 8.

On the calculator enter: $f(x,t) | x =$ _____ substitute the rule you obtained for x , in terms of t . Write down your expression then check it by using a range of values for t .

Answer: $\frac{2t^2}{11} - \frac{20t}{11} + 10$

Sample results: $\frac{2t^2}{11} - \frac{20t}{11} + 10 | t = 1$ $\frac{92}{11}$

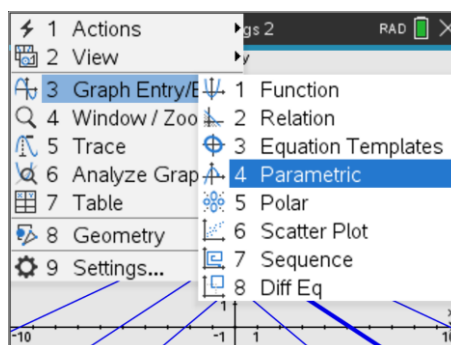
$\frac{2t^2}{11} - \frac{20t}{11} + 10 | t = 2$ $\frac{78}{11}$

$\frac{2t^2}{11} - \frac{20t}{11} + 10 | t = 3$ $\frac{68}{11}$

You now have an equation for x in terms of t and also one for y in terms of t . These are referred to as parametric equations.

The calculator is capable of graphing 'parametric equations'.

With the family of straight lines still in the Graphs Application change the graph type to parametric.



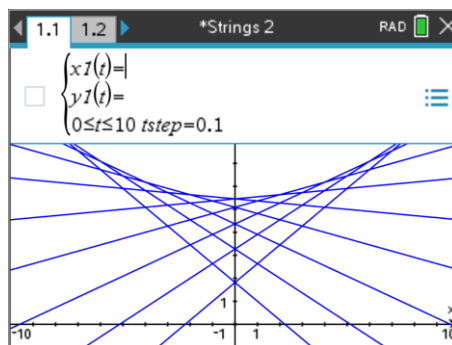
The equation entry now looks very different.

For the first equation, enter your equation for x.

For the second equation, enter your equation for y.

Change the range of values for t to go from 0 through to 10.

Change the t-step size to 0.1 (as shown).



Challenging Questions

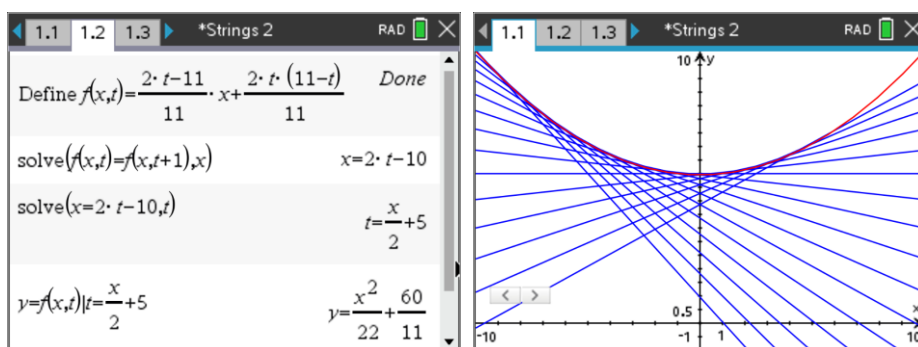
You have an equation for x in terms of t and y in terms of t.

Transpose the equation for x in terms of t to make 't' the subject of the equation.

Substitute your previous answer into the equation for y in terms of t. This leaves you with an equation of y in terms of x.

Graph your equation on the calculator (Graph Type: Function) to see if it also models the points of intersection between consecutive straight lines.

Discuss how well the equations model the 'curve' (envelope) produced by the straight lines.



Extension

it might seem a bit like cheating in Question 8, we had a difficult rule so we just used the solve command to work it out. The idea of a CAS enabled platform is to focus on problem solving rather than crunching numbers or following algorithms.

From time to time however the journey can be just as interesting as the destination. In other words, these calculations can be done 'by-hand' and there are a variety of ways to arrive at the same end result.

Dr. James Tanton has an easy to follow video on how to use 'difference tables' to determine an equation, such as the one generated in this task.



Watch Jame's video on YouTube:

<https://bit.ly/JTDifferenceTables>

Use the link or scan the QR code to watch James Tanton's video. Use his approach to determine a relationship between the equation pair (t) and the y coordinate.



Extension Answers: Most of this content is covered in Strings Part 4. Watch the video then download the student worksheet and teacher answers to Strings Part 4.