Continuity and Differentiability II

Math Objectives

- Students will explore piecewise graphs and determine conditions for continuity and differentiability.

Activity Type

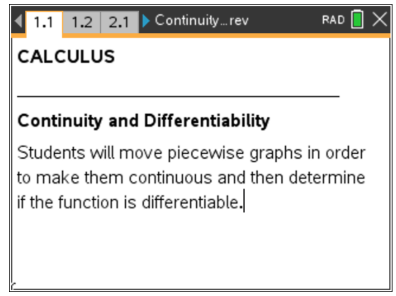
- Student Exploration

About the Lesson

- Students will change slider values on split screens of graphs. The left screen contains the graph of the piecewise function, and the right screen contains the graph of the derivative. This will allow students to see a visual representation of the algebraic process for determining continuity and differentiability.

Directions

- For each of the following problems, use the slider to change the value of *a* and/or *b* to make the function continuous and differentiable by examining the graphs. The graph on the right is derivative graph. Record your value for *a* or *b* then differentiate (by hand) to confirm that your values for *a* and *b* are correct. There may be more than one answer for a question. Students should attempt to solve the problem visually and then confirm answers algebraically.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Continuity_and_Differentiability_2_Student.pdf

Continuity_and_Differentiability_2_Student.doc

TI-Nspire document

Continuity_and_Differentiability_2.tns

Visit www.mathnspired.com for lesson updates.

Discussion Points and Possible Answers

PART I:

For each question write the function with the values of the slider variable(s) that make the function continuous and differentiable based on the graphs. There may be more than one answer. Confirm your solutions algebraically.

Problem 2.1:

Answer: $\mathbf{f}(x)=\begin{cases}-x, & x>-1\\ -x, & x<-1\end{cases}$ or $\mathbf{f}(x)=-x$

Problem 3.1:

Answer: $\mathbf{f}(x)=\begin{cases}-x, & x>1\\ -x^2, & x<1\end{cases}$

Problem 4.1:

Answer: $\mathbf{f}(x)=\begin{cases}x^2, & x>0\\ -2x^2, & x<0\end{cases}$

Problem 5.1:

Sample answers: $a-2=-4+b$

One possible solution: $\mathbf{f}(x)=\begin{cases}2x^2-2, & x>1\\ -4x^2+4, & x<1\end{cases}$

Another possible solution: $\mathbf{f}(x)=\begin{cases}x^2-2, & x>1\\ -4x^2+3, & x<1\end{cases}$

Problem 6.1:

Sample answers: $b=-a$

One possible solution: $\mathbf{f}(x)=\begin{cases}-2x^3+1, & x>-1\\ -x^3+2, & x<-1\end{cases}$

Another possible solution: $\mathbf{f}(x)=\begin{cases}x^3+1, & x>-1\\ -x^3-1, & x<-1\end{cases}$

Problem 7.1:

Answer: No value exists to create a continuous function.

Extension: Go back to each problem and determine if there is more than one solution that will result in a continuous function. If so, explain whether the additional solutions produce a

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differentiable or non-differentiable function.

Answer: Answers may vary.

PART II:

Calculate the derivative of each function using the values from part I and determine why the graph of the derivative is continuous or not. Explain your conclusion.

Problem 2.1:

Answer: $\mathbf{f}'(x) = -1$

Problem 3.1:

Answer: The graph is not differentiable because there is a cusp at $x = 1$.

Problem 4.1:

Answer: $\mathbf{f}'(x) = \begin{cases} 2x, & x > 0 \\ -4x, & x < 0 \end{cases}$

Problem 5.1:

Answer: The only value where $\mathbf{f}(x)$ is differentiable is when $a = -4$ and $b = -2$.

$$\mathbf{f}(x) = \begin{cases} -4x^2 - 2, & x > 1 \\ -4x^2 - 2, & x < 1 \end{cases}$$

$$\mathbf{f}'(x) = -8x$$

Problem 6.1:

Answer: The only value where **f**(x) is differentiable is when $a = -1$ and $b = 1$.

$$\mathbf{f}(x) = \begin{cases} -x^3 + 1, & x > -1 \\ -x^3 + 1, & x < -1 \end{cases}$$

$$\mathbf{f}'(x) = -3x^2$$

Problem 7.1:

Answer: The graph is not differentiable because the function is not continuous.

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