Math Objectives

- This file has a right triangle constructed in such a way that the lengths of the legs are always whole numbers.
- Students will drag the vertices and try to find whole number leg measures for which the measure of the hypotenuse is also a whole number.

Vocabulary

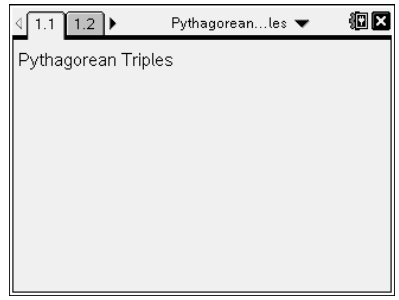
- Pythagorean Triples
- relatively prime

About the Lesson

- The time varies for this activity depending on whether the .tns file is provided to or created by the students.
- You may want to follow the directions to create the .tns file if you are planning on the students creating the file. If you are going to download the .tns file to the students' handhelds and let them begin with the file already constructed, you will need to investigate the way the .tns file works before you begin instruction.
- This activity assumes that students can find the greatest common divisor for a set of three whole numbers.
- Students might work independently or with a partner to complete this investigation.
- In this activity, Pythagorean Triples are defined as a set of three whole numbers that could be the lengths of the sides of a right triangle. Some texts may use *rational* instead of *whole* in this definition.
- You may want to use the term *relatively prime* when discussing Pythagorean Triples that have a greatest common divisor of 1 and connect this term to the word *primitive.*

TI-Nspire™ Navigator™ System

- Use Screen Capture to observe students' work as they proceed through the activity.
- Use Live Presenter to have a student illustrate how he or she used a certain tool.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Create Instructions
Pythagorean_Triples
_Create.pdf

Student Activity
Pythagorean_Triples
_Student.pdf
Pythagorean_Triples
_Student.doc

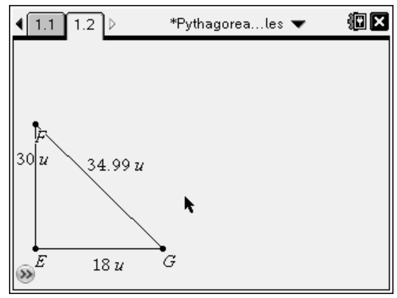
TI-Nspire document
Pythagorean_Triples.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the arrow until it becomes a hand ( ). Press ctrl to grab the point and close the hand ( ).

Move to page 1.2.



1. Drag points *F* and *G*. Observe the measures of the sides of this right triangle. Triangle *EFG* was created in such a way that the measures of the legs are always whole numbers.

   a. Do you think the measure that appears on the hypotenuse is exact or approximate? Explain your thinking.

   Answer: Sometimes it is exact and sometimes it is approximate. Using the Pythagorean Theorem to find the measure of the hypotenuse could produce an irrational number.

   b. If the sum of the square of the measures of the legs of a right triangle is a perfect square, what kind of number is the measure of the hypotenuse? Justify your answer.

   Answer: It would be a whole number. The square root of a perfect square is a whole number.

2. Drag points *F* and *G*. Try to find triangles where the measure of the hypotenuse is a whole number. Record the measures of the sides of these triangles in the table below.

   Sample Answers:

FE Leg Measure	*EG* Leg Measure	*FG* Hypotenuse Measure
6	**8**	**10**
10	**24**	**26**
20	**21**	**29**
7	**24**	**25**

Teacher Tip: This may take some time for students to do. You may want to have them share their triples with the class as they find them.

3. The triples of numbers you recorded in the table above are called Pythagorean Triples. A Pythagorean Triple is a set of three whole numbers that could be the measures of the sides of a right triangle.

   a. Select one of the triples from the table in question 2. What is the greatest common divisor (GCD) of these three numbers?

   Answer: Answers will vary.

   b. If the GCD is 1, then find the doubles of all three numbers. If the GCD is not 1, then divide each number by the GCD. Record the results below.

   Answer: Answers will vary.

Drag points *F* and *G*. Try to make a right triangle with these side measures.

4. If the GCD of the three whole numbers on a Pythagorean Triple is 1, then that triple is called a Primitive Pythagorean Triple. Primitive Pythagorean Triples can be used to generate other triples by multiplying a whole number scalar.

   Drag points *F* and *G* to find a Primitive Pythagorean Triple. Record it in the table below. Drag points *F* and *G* or use other methods to find other Pythagorean Triples generated by this Primitive Pythagorean Triple. Record these and the scalar that will produce each in the table below.

   Sample answers: Some answers (like the sample answers) will have only a couple of cases that can be viewed on the screen.

Scale Factor	*FE* Leg Measure	*EG* Leg Measure	*FG* Hypotenuse Measure
Primitive 1	**7**	**24**	**25**
2	**14**	**48**	**50**
3	**21**	**72**	**75**
4	**28**	**96**	**100**

Teacher Tip: This would be a great place to have the students prove that any multiple of a Primitive Pythagorean Triple is also a Pythagorean Triple. The two triangles are similar.

5. Which of the following are Pythagorean Triples? Justify each of your answers.

   a. 300, 400, 500

      Answer: This is a Pythagorean Triple; 90,000 + 160,000 = 250,000 (Pythagorean Theorem).

   b. 180, 181, 19

      Answer: This is a Pythagorean Triple; 32,400 + 361 = 32,761 (Pythagorean Theorem).

   c. 5, 5, 7

      Answer: This is not a Pythagorean Triple; 25 + 25 ≠ 49 (Contrapositive of Pythagorean Theorem).

   d. 60, 65, 25

      Answer: This is a Pythagorean Triple; 3,600 + 625 = 4,225 (Pythagorean Theorem).

6. For each Pythagorean Triple you found in question 5, give the Primitive Pythagorean Triple that generated it and the whole number scale factor that was used.

   Answer: 300, 400, and 500 was generated by 3, 4, 5; scale factor is 100.

   180, 181, and 19 is primitive; GCD is 1.

   25, 60, and 65 was generated by 5, 12, 13; scale factor is 5.

Extension—Generating Pythagorean Triples

Students might be interested in the following algorithm for generating Pythagorean Triples.

- Select two whole numbers, one even and one odd.
- Double the product of the two numbers selected. This product will be one element of the Pythagorean Triple representing a leg measure.
- Find the absolute value of the differences of the squares of the two numbers you selected. This number will be another element of the Pythagorean Triple representing the other leg.
- Find the sum of the squares of the two numbers you selected. This number will be another element of the Pythagorean Triple representing the hypotenuse.