Pebbling a Chess board

Student Activity

TI-Nspire™   Activity   Student   30 min

Introduction

This challenge is based on the mathematics problem created by Maxim Kontsevich, a very talented French and Russian mathematician. The problem involves getting three discs out of a specific region of a chess board. In this example we will consider the region to consist of the six shaded squares in the bottom left corner of the board. The rules of the challenge are as follows:

- Discs can only move forward (top of page) one space at a time.
- Each time a disc is moved it divides into two; one piece going forward and the other immediately to the right.
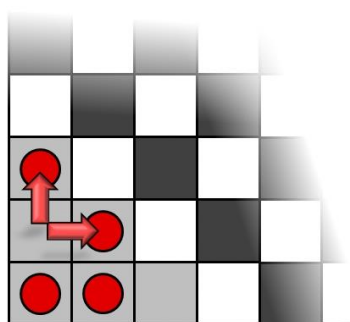- Discs cannot be placed on top of one another.
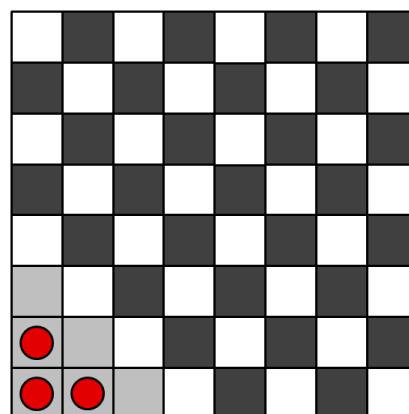
How many moves does it take to get all three pieces out of the shaded region?
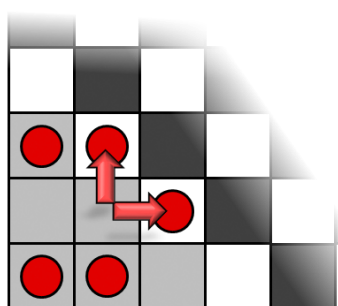
Equipment

- TI-Nspire Calculator
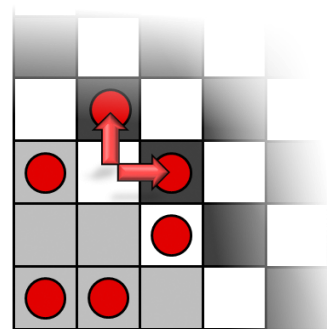- Chess Board & Counters[1]

Investigation

A possible first move.   A second move …   One more move and the first disc will be clear of the shaded region.

Question: 1.

What is the minimum number of moves required to leave just two discs in the shaded region?

Question: 2.

Estimate the number of moves required so that only one disc remains in the shaded region?

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[1] Numerous websites host a version of this challenge.

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Question: 3.

Using either a physical or digital platform, explore the problem and see if it is possible, within the confines of a chess board, to leave just one disc in the shaded region. Record your best attempt.

The problem becomes frustratingly difficult to free just two discs; however failure to succeed after numerous attempts does not mean a solution does not exist. It is possible to explore this problem theoretically using a numerical approach rather than relying on trial and error.

Suppose each counter is assigned a value. Consider the counters as shown:

Each time a counter is moved and splits into two new counters, the new counters are each worth half the original value. So when the very first half point counter is moved, the two new counters are worth a quarter of a point each.
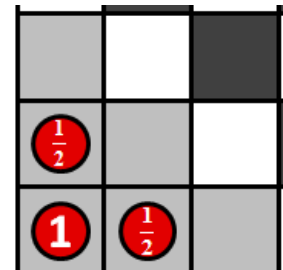
Question: 4.

What is the total value of the counters at the start of the problem?

Question: 5.

After one move, what is the total value of the counters in the problem?

Question: 6.

After two moves, what is the total value of all the counters in the problem?

Question: 7.

Imagine all the discs are now outside the grey area. What would be the sum of all the discs?

Question: 8.

Start the problem from the beginning and record in each square, the value of each counter in each corresponding square as the problem progresses.
Record your results on a blank chess board.

Question: 9.

Determine the sum of the values in the first 8 rows, **excluding** the shaded region, and also the sum of the values in the first 8 rows if the board could be extended indefinitely, **excluding** the shaded area.

Question: 10.

Determine the total value of the squares on the chess board outside the shaded region and comment on the result. **Hint**: What does this total need to be in order for the grey region to be empty?

Question: 11.

The problem may be easier if the board were made indefinitely large. Determine the total value of the squares outside the shaded region on a board of infinite size and comment on the result.

Question: 12.

If the shaded region were made smaller so that it was only just big enough to contain the original three discs and the board made infinitely large; the problem would appear much easier. Determine the number of moves required to remove all three discs from the new shaded region.

Question: 13.

Returning to the original chess board sized problem, imagine only two discs existed on the board at the start of the problem. Design a new, connected shaded region that would provide a challenge and would be solvable.

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TEXAS INSTRUMENTS