

# The Unit Circle

## Teacher Notes



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TI-Nspire CAS



Investigation



Student



60min

## Aim

The aim of this investigation is to use the unit circle to generate the graphs of sine, cosine and tangent functions.

## Equipment

For this activity you will need:

- TI-nspire CAS
- TI-nspire CAS documents – Sine Curve, Cosine Curve and Tangent Curve

**National Curriculum Statement:** Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies (ACMMG274)

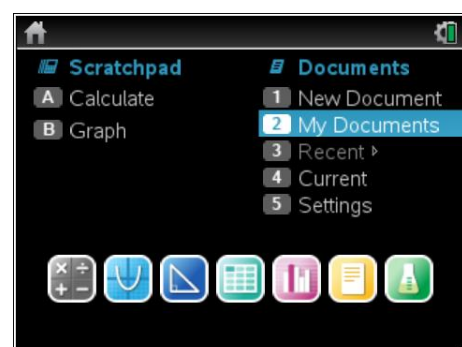
## Introduction – Setting up the calculations

During this activity, students will need use three TI-nspire files: “Sine Curve”, “Cosine Curve” and “Tangent Curve”. These files can be distributed using TI-Navigator, the TI-nspire docking station or the teacher/student software. To distribute the files using the Teacher software, use the **Tools** menu and select the **Transfer Tool**. Locate the TI-nspire files on your computer and then start the transfer. Once the files are transferred to the first handheld, unplug the handheld and continue plugging in each student’s handheld device. Once all the students have the files, stop the transfer. Note that students can also transfer the files from one handheld device to another from within the **My Documents** folder. Note also that multi-port USB connectors can be used to transfer files to several handhelds at the one time.

This activity requires access to the “Sine Curve”, “Cosine Curve” and “Tangent Curve” TI-nspire documents. These documents should be loaded on your device before proceeding.

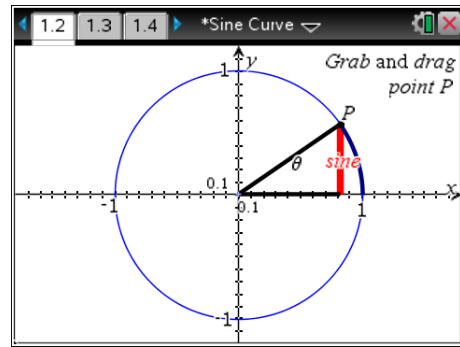
Once the document is on your handheld, press **home** and select **My Documents**. Locate the “Sine Curve” document and press **enter** to open.

The location of the file depends on the selected location during the file transfer.



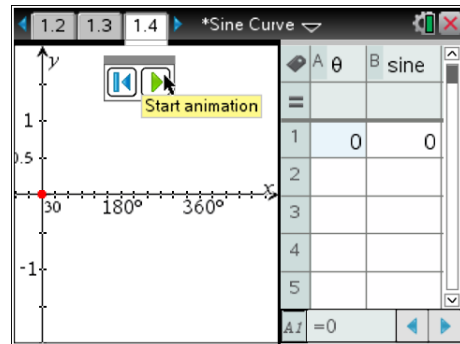
Navigate to page 1.2, then grab and drag point P around the unit circle. Observe how the vertical line segment representing the **sine** changes as the point P is dragged around the unit circle.

Next, we will graph these values of the sine function.



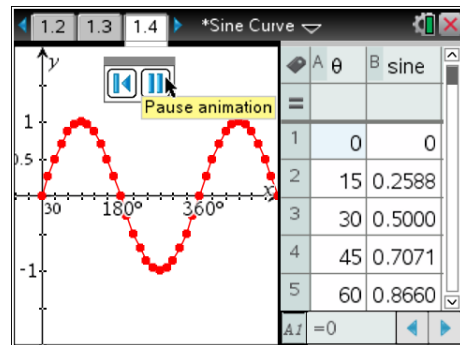
Navigate to page 1.4 and press the **start** button on the motion controller.


Ensure that the students pause the animation as it will eventually reset all values. The animation can be reset at any stage by clicking on the reset button.



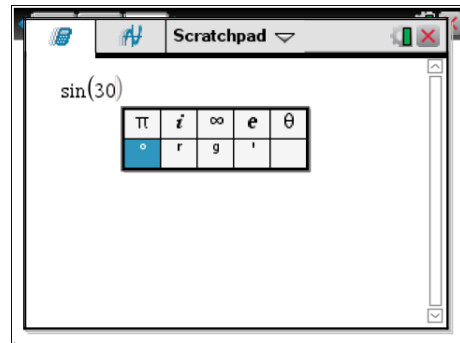
Once the sine curve has been drawn, press the **pause** button on the motion controller to stop the animation. The graph of the sine function will be drawn on the left. The values of the angle and the corresponding sine of the angle will be displayed on the right.

To swap between windows, use the **mouse** to click on a window or press **ctrl + tab**.





Part of in this investigation requires calculations to be performed. The Scratchpad is a place where calculations can be computed and then discarded. To access the Scratchpad press **home** and select **Scratchpad** (or press **A**). Alternatively, press the  key (this key is not available on a Clickpad).

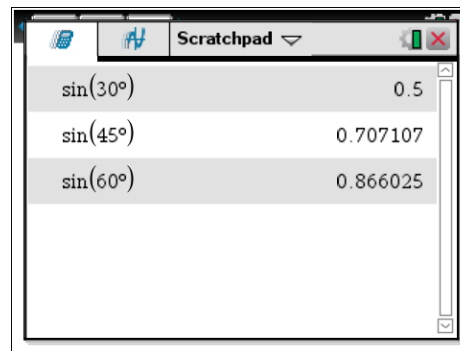
Confirm some of the sine values from the table.



It is recommended that the settings be changed to **Degree** and **Approximate**.

Change the document setting to **Degree** or use the degree symbol (°). This can be found in the symbol palette or by pressing  or  to access one of the mini-palettes. On a Clickpad, press **ctrl + °**.

To find the approximate answer, change the document setting to **Approximate** or press **ctrl +**



**enter.**

To return to the current document, press **esc**. If you are on the home page, press **Current** (or press **4**).

1. Copy and complete the table below by selecting five values from the table on page 1.4. Compare them to your own calculations using the Scratchpad (see example below).

Several values are possible for the following tables.

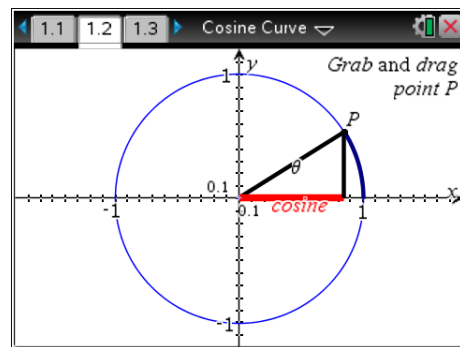
angle ( $\theta$ )	$45^\circ$	$60^\circ$	$120^\circ$	$195^\circ$	$270^\circ$	$330^\circ$
sine (from table)	0.7071	0.8660	0.8660	-0.2588	-1.000	-0.5000
$\sin(\theta)$ (from Scratchpad)	0.707107	0.866025	0.866025	-0.258819	-1.	-0.5

2. In the table on page 1.4, you will notice that many of the sine values are repeated. Copy and complete the table below by selecting five pairs of equal sine values (see example below).

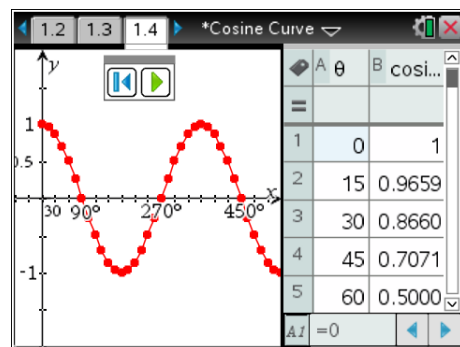
sine	0.7071	0.5000	0.8660	-0.2588	-0.5000	-0.8660
$\theta$ (first angle)	$45^\circ$	$30^\circ$	$60^\circ$	$195^\circ$	$210^\circ$	$240^\circ$
$\theta$ (second angle)	$135^\circ$	$150^\circ$	$120^\circ$	$345^\circ$	$330^\circ$	$300^\circ$

Press **home** and select **My Documents**. Locate the “Cosine Curve” document and press **enter** to open.

Navigate to page 1.2, then grab and drag point P around the unit circle. Observe how the horizontal line segment representing the **cosine** changes as the point P is dragged around the unit circle.



Navigate to page 1.4 and press the **start** button on the motion controller. The graph of the cosine function will be drawn on the left. The values of the angle and the corresponding cosine of the angle will be displayed on the right.



3. Copy and complete the table below by selecting five values from the table on page 1.4. Compare them to your own calculations using the Scratchpad (see example below).

angle ( $\theta$ )	$30^\circ$	$45^\circ$	$90^\circ$	$105^\circ$	$225^\circ$	$300^\circ$
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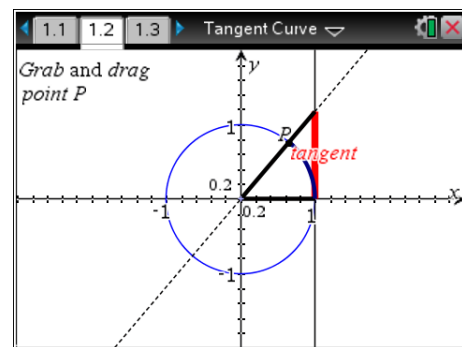
cosine (from table)	0.8660	0.7071	0.0000	-0.2588	-0.7071	0.8660
$\cos(\theta)$ (from Scratchpad)	0.866025	0.707107	0.	-0.258819	-0.707107	0.866025

4. In the table on page 1.4, you will notice that many of the cosine values are repeated. Copy and complete the table below by selecting five pairs of equal cosine values (see example below).

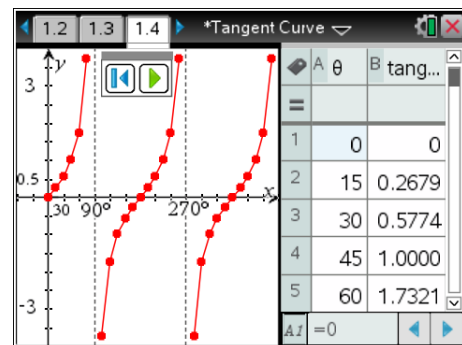
cosine	0.8660	0.7071	0.5000	-0.7071	-0.8660	-0.9659
$\theta$ (first angle)	$30^\circ$	$45^\circ$	$60^\circ$	$135^\circ$	$150^\circ$	$165^\circ$
$\theta$ (second angle)	$330^\circ$	$225^\circ$	$300^\circ$	$225^\circ$	$210^\circ$	$195^\circ$

Press **home** and select **My Documents**. Locate the “Tangent Curve” document and press **enter** to open.

Navigate to page 1.2, then grab and drag point P around the unit circle. Observe how the horizontal line segment representing the **tangent** changes as the point P is dragged around the unit circle.



Navigate to page 1.4 and press the **start** button on the motion controller. The graph of the tangent function will be drawn on the left. The values of the angle and the corresponding tangent of the angle will be displayed on the right.



5. Copy and complete the table below by selecting five values from the table on page 1.4. Compare them to your own calculations using the Scratchpad (see example below).

angle ( $\theta$ )	$30^\circ$	$15^\circ$	$60^\circ$	$150^\circ$	$225^\circ$	$285^\circ$
tangent (from table)	0.5774	0.2679	1.7321	-0.5774	1.0000	-3.7321
$\tan(\theta)$ (from Scratchpad)	0.57735	0.267949	1.73205	-0.57735	1.	-3.73205

6. In the table on page 1.4, you will notice that many of the tangent values are repeated. Copy and complete the table below by selecting five pairs of equal tangent values (see example below).

tangent	1.7321	0.5774	3.7321	#UNDE...	-0.5774	-0.2679
$\theta$ (first angle)	$60^\circ$	$30^\circ$	$75^\circ$	$90^\circ$	$150^\circ$	$165^\circ$
$\theta$ (second angle)	$120^\circ$	$210^\circ$	$255^\circ$	$270^\circ$	$330^\circ$	$345^\circ$

7. Describe the shape and key features of the sine, cosine and tangent graphs. Include features such as the initial values, the maximum and minimum values, the x-intercepts and any asymptotes (breaks in the graph).

Sine curve – The graph starts at 0, goes up to a maximum value of 1 and then goes down to a minimum value of  $-1$ . It has x-intercepts every  $180^\circ$ , at  $0^\circ$ ,  $180^\circ$ ,  $360^\circ$ , etc. The graph completes a full cycle every  $360^\circ$ .

Cosine curve – The graph starts at 1, goes down to a minimum value of  $-1$  and then comes back up to a maximum value of 1. It has x-intercepts every  $180^\circ$ , at  $90^\circ$ ,  $270^\circ$ ,  $450^\circ$ , etc. The graph completes a full cycle every  $360^\circ$ .

Tangent curve – The graph starts at 0, goes up to infinity at  $90^\circ$ . It then starts at negative infinity and increases back to 0. The graph has asymptotes every  $180^\circ$ , at  $90^\circ$ ,  $270^\circ$ ,  $450^\circ$ , etc. The graph completes a full cycle every  $180^\circ$ .

8. Complementary angles are pairs of angles that add to  $90^\circ$ . For example,  $30^\circ + 60^\circ = 90^\circ$  so  $30^\circ$  and  $60^\circ$  are complementary angles. The cosine function is the complement of the sine function. The short way of writing 'complement of sine' is **cosine**. Therefore  $\sin(30^\circ) = \cos(60^\circ)$ . Copy and complete the table below by finding the sine and cosine values of five pairs of complementary angles (see example below). Round your answers to four decimal places.

complementary angles	$30^\circ/60^\circ$	$10^\circ/80^\circ$	$15^\circ/75^\circ$	$23^\circ/67^\circ$	$32^\circ/58^\circ$	$40^\circ/50^\circ$
$\sin(\text{first angle})$	0.5	0.1736	0.2588	0.3907	0.5299	0.6428
$\cos(\text{second angle})$	0.5	0.1736	0.2588	0.3907	0.5299	0.6428

Note – On each of the three files, page 1.5 contains a program for the motion controller on page 1.4. It is not intended that students view page 1.5.