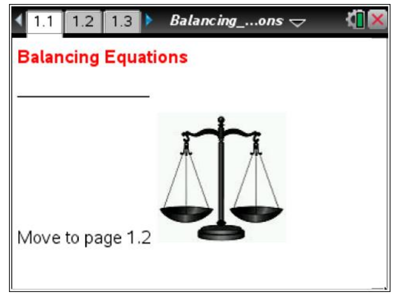
Balancing Equations

MATH NSPIRED

TEACHER NOTES



Math Objectives

- Students will solve linear equations in one variable graphically and algebraically.
- Students will explore what it means for an equation to be balanced both graphically and algebraically.

Vocabulary

- equation
- variable
- balanced equation
- equivalent equation
- transforming an equation
- simplifying an expression
- addition/subtraction/multiplication/division property of equality

About the Lesson

- This lesson involves understanding what it means for an equation to be balanced in the process of solving linear equations with one variable. As a result, students will:
  - Examine the graphs of the two separate expressions from a linear equation with one variable and then noticing their point of intersection as a solution to the original equation.
  - Find the solution(s) of a linear equation by transforming it into equivalent equations in simpler forms using the idea of balancing equations and the properties of equality.
  - Determine if a linear equation has one, infinitely many, or no solutions by examining the final equivalent equation found.

TI-Nspire™ Navigator™

- Use Live Presenter to have students explain their strategies for algebraically solving linear equations with one variable.
- Use Quick Poll to assess students' understanding.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Balancing_Equations_Student.pdf
- Balancing_Equations_Student.doc

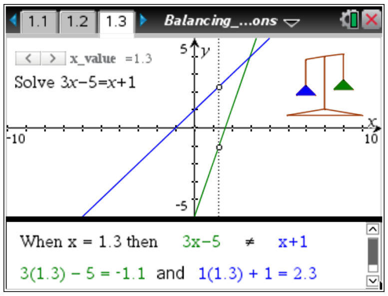
TI-Nspire document

- Balancing_Equations.tns

Discussion Points and Possible Answers

Teacher Tip: The objective 'examine the graphs of the two separate expressions from a linear equation with one variable and then noticing their point of intersection as a solution to the original equation' is to provide for a visual representation for balancing equations. The expressions from the right- and left-hand sides of the equation are being treated as separate functions so they can be represented graphically. This representation is not intended as a *technique* for students to use to solve linear equations but rather to promote conceptual understanding in balancing equations.

Move to page 1.3.



1. Before you begin balancing equations, describe the various items you see on Page 1.3.

   Sample Answers: I see a graph with two colored lines and a point on both lines. I see an unbalanced scale. I see some equations and a slider next to the words **x value**. I see "Solve $3x - 5 = x + 1$".

2. Click on the slider to change the value of *x.*

   a. Describe what happens when you change the value of *x*. What happens with the points on the lines? What happens to the balance? What happens with the equations at the bottom of the screen?

      Sample Answers: When I clicked on the slider, I could make the *x*-value bigger or smaller. When I made the *x*-value smaller, the points moved down the lines and when I made the *x*-value bigger, the points moved up the lines. The balance moved up and down as the *x*-value changed. Most of the time, the equation at the bottom of the screen said $3x - 5 \neq x + 1$ since both sides were not equal to the same value. For example, when x was 2.6 then 3*(2.6) – 5 = 2.8 and 2.6 + 1 = 3.6.

b. How are the changes you described in part a related?

Sample Answers: Changing the *x*-value moved the points, changed the values in the equations, and moved the balance all at the same time. When x was 3, I finally had both points on the lines on top of each other. It said I had found a solution, the scale was balanced, and both sides of the equation were equal to 4.

> **Teacher Tip:** Many things are happening on Page 1.3 when students explore changing the value of *x*. (If students have not used a slider before, direct their attention to the slider box, and indicate that this is referred to as a **slider**. The slider controls the values for *x* on the page. They need to move the cursor to the slider controls, click to get a pointing finger, and then click on either the right or left side of the Touchpad.) Encourage students to examine all things that are changing: the points on the graphs, the balance, and the various equations.
> In this activity, we are focusing on balancing a single equation. To model this idea, we are graphing each *side* of the equation as a line. Students can track what happens on each side separately and visually as they substitute values of *x* in the original equation. They should see that when $x \neq 3$, the equation, $3x - 5 = x + 1$, is not a true statement. When $x = 3$, they have found a solution to the equation, both sides of the equation have the same value, the lines intersect, and the scale is balanced. Explorations on this page are designed to help promote a conceptual sense of what is happening when students balance and solve linear equations.

c. What values for *x* make the equation true?

Answer: $x = 3$.

d. How many values of *x* make the equation true? How does the graph support your answer?

Answer: Only one *x*-value makes the equation true because there is only one intersection point between the two lines. Only one value makes both sides of the equation produce the same value.

> **Teacher Tip:** Emphasize that both sides of the equation have the same value, 4, when $x = 3$, which only happens once. Having two lines intersect at only one place supports the idea that there is only one solution that balances the equation, making both sides of the equation (expressions) have the same value and making the equation a true statement.

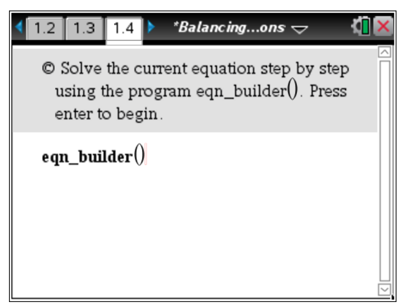
TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

e. For what values of x is the left-hand expression less than the right-hand expression? How does the graph support your answer?

Sample Answers: When $x < 3$, then values for the expression $3x – 5$ are less than values for $x + 1$. I can see that because the points on the line for $3x – 5$ ($y = 3x – 5$) are beneath the points on the line for $x + 1$ ($y = x + 1$).

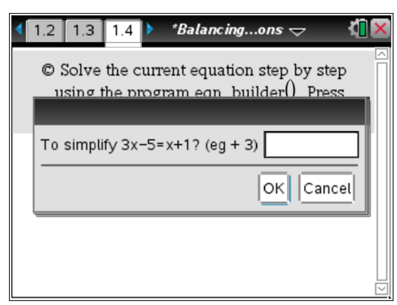
Teacher Tip: This question has students notice what is happening to the values for each expression separately as the x-value changes. You can ask the companion question: For what values of x is the left-hand expression *greater* than the right-hand expression? Students should notice this is true when $x > 3$. Something interesting happens when $x = 3$; both sides of the equation finally have the same value. On either side of $x = 3$, the values are not equal and visually grow farther apart, helping support their response in part d that there is only one solution to the equation in this example.

Move to page 1.4.



3. Press [enter] to solve the equation $3x – 5 = x + 1$.

Tech Tip: To access the keyboard, tap the middle of the page.



- To find a solution to the equation algebraically, you want to change $3x – 5 = x + 1$ to a simpler, but equivalent form so that eventually your equation is $x = a$, where a is the solution.
- As you make the changes to the original equation, $3x – 5 = x + 1$, be sure the new form of the equation is equivalent to the original. Another way to think about this is to keep both sides of the equation “balanced” as you make your changes, so that the transformed equation is also a true statement for the same value of x.
- You will use the addition, subtraction, multiplication, and division Properties of Equality to transform your equation until it appears in its simplest form “$x = a$”.

a. In the pop-up screen, you are asked to provide an operation (addition, subtraction, multiplication, or division) and a numeric value or a variable. Whatever you enter will be done to **both sides** of your equation to maintain the balance. If your final goal is to change $3x - 5 = x + 1$ into the simpler form $x = a$, you might want to try adding 5 to both sides of the equation. Predict what changes will happen to the equation if you add 5 to both sides.

Sample Answer: The equation will change to $3x = x + 6$.

> **Teacher Tip:** The program in this document allows students to create equivalent equations. This allows students to experiment algebraically with what they just explored visually on Page 1.3, that of balancing equations and finding solutions. Students can experiment with choices in making equivalent equations and then decide if they made efficient choices in getting to a final goal of $x = a$. If students do not choose a more efficient route, it can provide an opportunity for class discussion on how individuals are making their choices.

> **Tech Tip:** If at any time the student wants to exit the equation builder program, they just need to enter a 0 into the dialog box.

b. After you have made your prediction, enter + 5 into the box, press enter. Was your prediction correct?

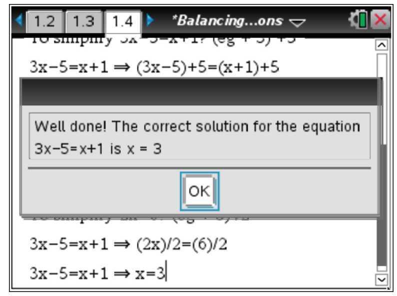
Sample Answers: Yes.

c. You are getting closer to your goal of creating a simpler equation of $x = a$. What do you want to try next? Discuss your decision with a partner to convince him/her that you are making an **efficient** choice. In other words, your choice will help you find a solution in as few steps as possible.

Sample Answer: I will subtract x from both sides so that all of the xs will be on one side of the equation and the other side will just have a number.

d. Once you and your partner have agreed on a decision, enter the operation into the box, press enter, and verify that your thinking is appropriate.

e. Continue with this process of deciding what to do next, convincing your partner of your thinking, and entering your decision until you finally arrive at an equation of $x = a$. Your final screen will tell you that you have found the correct solution.



Answer: See screen shot.

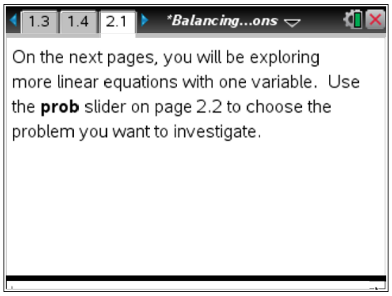
Teacher Tip: When students have finished finding a solution to the equation, their thinking has been saved on the page. Students could share their process individually by using a document camera or this could be another **TI-Navigator Opportunity** using ***Class Capture*** **or** ***Live Presenter*** (**see Note 2 at the end of this lesson**).
Scrolling through the Calculator page emphasizes the process that occurs when transforming an equation. First, apply one of the properties of equality to both sides; second, simplify each side. Keep repeating this cycle until the equation has been transformed to either $x = a$ or $a = x$.

f. Does the final solution you found algebraically match the solution you found graphically? Should it? Explain your thinking.

Answer: Yes, my solution matches. It should match because the value of x for the intersection point, 3, is the value that makes the equation a true statement: $3(3) – 5 = 3 + 1$.

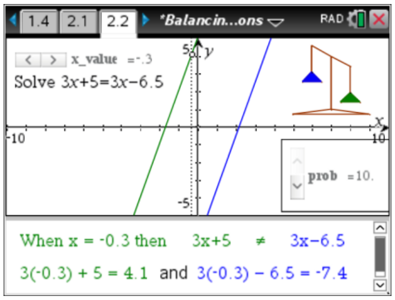
Tech Tip: Students can reuse the equation builder program on Page 1.4 by using the up arrow until the line that reads “eqn_builder()” is highlighted, and then pressing [enter] to paste this program command on the active line of the Calculator page. Finally, move the cursor to the left one space so that the cursor (a vertical bar) appears between the two parentheses, and press [enter]. Students can explore solving this same equation with the equation builder in a variety of ways, including ways that might not be quite as efficient as others, either because of the increased number of steps involved or because of the difficulty of the calculations during the simplify step. For this equation, students could try “- x” first, then “+ 5”, and finally “/ 2”. Both strategies took 3 steps, so both are just as efficient.

Move to page 2.1.



4. Now you will be asked to explore several other equations, first using a visual approach and then using an algebraic approach. Use the ***prob*** slider on the Page 2.2, and press ▲ or ▼ to create a new equation. Find a solution to the new equation both graphically (on Page 2.2) and algebraically (on Page 2.3). Write below how you found your solution algebraically for several different equations.

   <u>Sample Answers</u>: Student answers will vary.



Teacher Tip: There are nine linear equations. After students solve an equation graphically on page 2.2, they can solve the equation algebraically on page 2.3, and then return to page 2.2, and use the ***prob*** slider to select another equation.
Note that the equation for problem 10 is the focus of question 5 in the Student Worksheet. This equation is an example of an equation that has no solution and should promote an interesting discussion as a whole class.

2. $5x - 6 = x - 4$	**3**. $-x - 7 = x - 1$	**4**. $3x + 5 = -2x + 1$
5. $-2x + 6 = 4x - 9$	**6**. $4x + 7 = -x + 1$	**7**. $-7x + 2 = -2x - 2$
8. $-x + 1 = -6x - 9$	**9**. $5x + 3 = -5x - 10$	**10**. $3x + 5 = 3x - 6.5$.

5. Use the ***prob*** slider on Page 2.2, and press ▲ until you are at the last equation, $3x + 5 = 3x - 6.5$.
   a. What is different about the graph of this example from all of the other equations you have solved? What is different about the final equation algebraically?

      <u>Sample Answers:</u> This graph has two parallel lines instead of two intersecting lines. When I solved the equation, my final equation was $0 = -11.5$ (or $11.5 = 0$ or $5 = -6.5$). It has no solution.

Teacher Tip: Ask students how they are interpreting their final equation algebraically. For $0 = -11.5$, for example, be sure that they understand that this is the same as $0x = -11.5$. None of the results ($0 = -11.5$ or $11.5 = 0$ or $5 = -6.5$) are true statements; they are all false. Therefore, there is no solution to this linear equation. Graphically, they should see there is no point of intersection, also indicating there is no solution to the equation.

b. Move the *x* slider to find a solution to this equation. What do you notice? Why is this happening?

Sample Answers: There is no point of intersection since the two lines are parallel.

c. Write another equation for which you think the same thing would happen.

Sample Answer: $2x + 5 = 9 + 2x - 2$.

> **Teacher Tip:** Ask students if they notice any pattern in the algebraic equations where there is no solution. Since the coefficients of x on both sides of the equation are the same but the values of the constants are not, the graphs of the equations ($y = 2x + 5$ and $y = 9 + 2x - 2$) are parallel.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 3 at the end of this lesson.

Extension:

6. Solve the following equation: $4x + 8 - x = 15 + 3x - 7$. What is your solution?

Sample Answers: $3x + 8 = 3x + 8$. When I try to solve for *x*, I get $0 = 0$. Both sides of the equation are the same, so the graph would be only one line. There are a lot of values for *x* that make the equation a true statement.

> **Teacher Tip:** Ask students if they notice any pattern in the algebraic equations when there are an infinite number of solutions. The expressions on both sides of the equation are identical. If each side of the equation is set equal to y and graphed, the graphs are the same. You can also choose not to examine this problem at this time, dependent upon the progress of the students' understanding during the activity.

7. Do all linear equations have exactly one solution? Explain your thinking.

Sample Answers: No. We solved one in Problem 5 that had no solution and in Problem 6, there were many solutions to the linear equation.

Wrap Up

Upon completion of the activity, the teacher should ensure that students are able to:

- Describe what it means to maintain a balanced equation when they transform or simplify an equation.
- Solve a linear equation with one variable by creating equivalent equations in simpler form.
- Determine if a linear equation has one, infinitely many, or no solutions by examining the final equivalent equation found.

Assessment

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*
See Note 4 at the end of this lesson.

1. Is $2x - 15 = 1$ an equivalent equation to $9x - 15 = 7x + 1$?
2. In solving the equation $9x - 15 = 7x + 1$, a student wrote the following: $9x - 15 + 15 = 7x + 1 - 1$. Is this new equation equivalent to the original?
3. Is $x = 8$ a solution to the equation $9x - 15 = 7x + 1$?
4. How many solutions are there to the equation $9x - 15 = 7x + 1$?
5. Write a linear equation using one variable that would have no solutions.
6. Write a linear equation using one variable that would have an infinite number of solutions.*

> ***Teacher Note**: You might not want to ask #6 in the assessment if you did not have time to fully develop the idea of an infinite number of solutions.

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Note 1

Question 2c and 2d, Quick Poll

This is a possible moment to use *Quick Poll.* Allow for a brief amount of time for students to use the slider and watch what happens with the points on both lines, the movement of the scale balance, and the values for both sides of the equations. *Quick Poll* could be used to see the solutions students are finding as well as the number of solutions they find and encouraging discussion to support their responses.

Note 2

Question 3e, Class Capture or Live Presenter

If you use Class Capture, you could examine all the strategies students used to find their final solution of $x = a$, and discuss which ones were more efficient and why. Live Presenter could also be used and might be more effective since the entire page of work is more than a single screen shot; with Live Presenter, a student could scroll through his/her Calculator page.

Note 3

Question 5c, Class Capture

Use Class Capture to have the class examine all equations they believe have no solutions to find a pattern to predict when an equation would have no solution.

Note 4

Assessment, Quick Poll

A Quick Poll can be given at the conclusion of the lesson to get a sense of student understanding. You can save the results and display students' responses at the start of the next class to discuss possible misunderstandings students might have.