Global Greatness - Latitude

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Answers & Teacher Notes

7 8 9 10 11 **12**

TI-Nspire™ | Investigation | Student | 60 min

Aim

To determine the distance along the surface of the Earth between two places that are on the same meridian of longitude.

Equipment

For this activity you will need:

- TI-Nspire CAS
- Ti-Nspire CAS documents:
  - Worldsphere.tns
  - World distance.tns

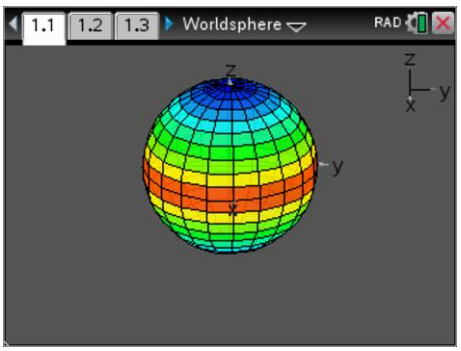
Introduction

Which city is closer to the equator, Darwin or Cairns? It may surprise you to know that the shortest distance between two points on the Earth lies along a curve, not a straight line. Our planet is approximately spherical having a radius of 6400km. Explore a desktop globe, some software or Apps such as Google Earth. You can build a virtual model using the 3D Graphing feature on TI-Nspire.

The Global Sphere

Open the TI-Nspire file Worldsphere.tns. The sphere that you see can be thought of as a representation of the Earth. The z-axis passes through the north and south pole and represents the Earth's axis of rotation.

The x and y axis plane slice through the equator.



Note: Pages 1.2 to 1.5 contain copies of questions 1 to 5 below.

Press the letter **A** on the calculator keypad to see the sphere automatically rotate, just as the Earth rotates on its axis. (Press [ESC] to stop the rotation) The sphere can also be rotated manually by grabbing any region off the sphere.

Question: 1

What colour on the sphere are the north and south poles? Blue

Question: 2

What colour on the sphere is the equator? Orange

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Question: 3

The lines that run across the sphere from left to right represent:

(A) – Meridians of longitude

(B) – Parallels of latitude

Question: 4

The lines that run between the north and south pole on the sphere (up/down) represent:

(A) – Meridians of longitude

(B) – Parallels of latitude

Question: 5

The direction of the rotation indicates that the Earth spins:

(A) – Towards the East

(B) – Towards the West

Case 1: Calculating distance along a meridian using different latitudes north of the equator

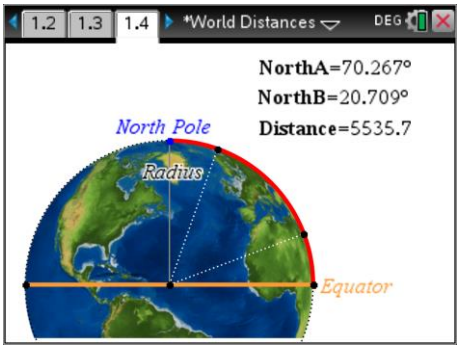
Open the TI-Nspire file World distance.tns, read the introductory pages and navigate to page 1.4.

The diagram on this page is a side-on view of the Earth.

Red line: Meridian line (north of the equator)

Orange line: Equator

Pink line: Arc connecting two locations on the same meridian but different latitude, north of the equator.



Grab these points and move them to observe the changing latitude and the distance. (Distance n kilometres)

The distance computed on Page 1.4 uses an extension of the arc length formula:

$$s = r \times \frac{\pi}{180} \times \theta^{\circ}$$ **–- Equation 1**

The formula being used by the calculator is:

$$s = 6400 \times \frac{\pi}{180} \times |northa - northb|$$ **–- Equation 2**

Question: 6

What is the meaning of the figure '6400' in equation 2? 6400 is the approximate radius of Earth.

Question: 7

What is the meaning of |northa – northb| in equation 2?

This expression determines the 'difference between two latitudes'. The absolute value signs produce a positive result regardless of whether northa > northb or northb > northa.

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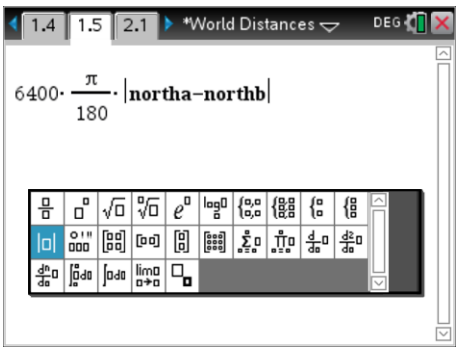
Navigate to page 1.5 and type out the formula.



Note: The variables northa and northb can be obtained by selecting the [VAR] key.

The absolute value sign is available from the maths template.

The result of this calculation shows the distance in km between the two points. Check your answer against the automatic answer produced on page 1.4

Question: 8

Mobile [Alabama] 30.695°N and Chicago [Illinois] 41.878°N are both located close to the 88°W meridian in the northern hemisphere.

a) Move points A and B on Page 1.4 to angles that approximate these locations and record the corresponding approximate distance.
Answers will vary depending on the accuracy of point placement. Typical value: 1242km

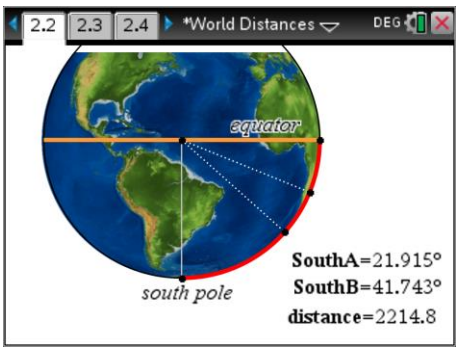
b) Use the formula (equation 2) replacing NorthA and NorthB with the actual latitudes to obtain a more precise result for the distance between Mobile and Chicago.
Using above latitudes: 1249.15km

c) Use Google Maps to determine the distance between Mobile and Chicago[1].
Google Maps: 1355km (Using the walking option provides the most direct route.)

d) Discuss the differences between the three answers (a), (b) and (c).
Option A is approximate due to the approximate angles used.
Option B provides a more accurate answer but it assumes the locations are exactly on the meridian (Great Circle).
Option C uses actual paths rather than 'as the crow flies'.

Case 2: Calculating distance along a meridian using different latitudes south of the equator

The diagram on page 2.2 is a side-on view of the Earth. In this interactive diagram points A and B are located on the same meridian south of the equator.



$$s = 6400 \times \frac{\pi}{180} \times |southa - southb|$$ **–- Equation 3**

[1] **Google Maps** returns distances in 'miles' for locations within the United States. To convert miles into metres on the calculator use _mi after the distance, the calculator will automatically return the distance in metres provided your calculator mode is set for SI units.

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Question: 9

Melbourne [Victoria] 37.8136°S and Cairns [Queensland] 16.9186° S are both located adjacent to the 145°E meridian in the southern hemisphere.

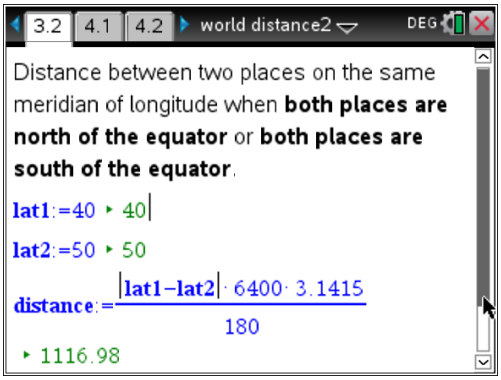
a) Move points A and B on Page 2.2 to angles that approximate these locations and record the corresponding approximate distance.
Answers will vary depending on how accurately the angles are obtained: 2342km

b) Use the formula (equation 2) replacing SouthA and SouthB with the actual latitudes to obtain a more precise result for the distance between Melbourne and Cairns.
Using the angles above: 2334km.

Case 1 & 2: Calculating distance along a meridian using different latitudes in the same hemisphere.

On page 3.2, edit the latitude values that are in blue immediately following the = signs for latitude 1 (Lat1) and latitude 2 (Lat2). Press Enter after each edit and your value will be confirmed in the green text.

The formula will automatically recalculate using your values and the final output (in green) gives the distance (km) between the two points on the same meridian.



The automatic calculations on this page are achieved using a "Maths Box" contained on a Notes Application.

Question: 10

Portland, Oregon (USA) is almost due south of San Francisco, California. Portland is located at 45.5231° N, 122.6765° W. San Francisco is located at 37.7749° N, 122.4194° W.

a) Which measurement for the location of Portland and San Francisco represents the longitude? (Meridian) Both cities are very close to the 122°W meridian.

b) Use the automatic calculation feature on page 3.2 to determine the approximate distance between Portland and San Francisco. 865.5km using above latitudes.

c) Explain, with reference to the locations, why the calculation is approximate.
Locations are close to meridian but not exact, therefore distance is approximate.

Question: 11

Bendigo (Victoria) is located at: 36.7570°S, 144.2794°E. Geelong (Victoria) is located at 38.1499°S, 144.3617°E.

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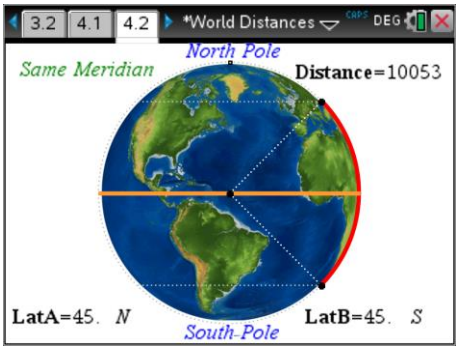
a) Determine the approximate distance between Bendigo and Geelong.
155km using above latitudes.

b) A sign post just outside Geelong shows the distance to Bendigo. Explain the difference between the calculated result and the sign post.
The sign post is providing road distances which include the small deviations to go into townships, around corners whereas the computed distance is 'as the crow flies'.

Case 3: North and South.

A globe is shown on page 4.2. The two points can move around the globe along the same great circle. The red arc continues to show the shortest distance between the two locations, however allowances must be made when a combination of North and South locations are in the same problem.

The light grey dotted lines parallel to the equator represent other locations with the same latitude.



Question: 12

In the previous diagram (Page 4.2), location A is at latitude 45° in the northern hemisphere and location B is at latitude 45° in the southern hemisphere.

a) If 45° was used for both locations, what result would Equation 2 produce? 0km (Zero)

b) What is the angle between the two locations and how could the angle(s) be 'modified' to make the calculation work? (Justify your answer.)
The actual angle would be 90°. By making 'south' measurements negative and 'north' positive the difference will be measured correctly. This can be reversed with north as negative and south as positive since the absolute value computation will remove the negative sign resulting in either computation.

Note: Students may provide sample calculations as justification.

Question: 13

Perth is located at 31.95° S, 115.9°E. Beijing is located at 39.90° N, 116.4°E.

a) To which meridian are Perth and Beijing closest? 116 E.

b) Use the diagram on Page 4.2 to estimate the distance between Perth and Beijing.
Answers will vary due to approximate angles: 8000km

c) Use your 'modified' bearings and an appropriate formula to determine a more accurate distance between Perth and Beijing.
Using the bearings: 8025km

Comments:

- Using Google Earth the distance is 7985km, the difference can be attributed to the two locations either side of the meridian. There are numerous websites that allow the user to enter both longitude and latitude for both locations. This can be demonstrated to

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students to show them that the computations are approximate when the arc length formula is used as it assumes the two locations lie on a great circle.

Useful Definitions

Radius:	As the radius of a circle is the distance from the centre of a circle to any point on the circle, the radius of the Earth is the distance from the centre of the Earth to any point on the surface of the Earth (at sea level). This is approximately 6400 km.
Great Circle:	Imagine slicing the Earth exactly into two hemispheres so that the slice goes through the very centre of the Earth. The circular faces that would be made are called *Great Circles*. We can actually consider a great circle as the pathway that this makes across the surface of the Earth. (There's really no need to chop the Earth in half, as that would be very messy!) The equator is one example of a great circle. Airline flight paths also generally follow great circle arcs across the globe, because that is the shortest way to get between two points on the Earth's surface.
Small Circle:	If you made a planar slice through the Earth, but didn't slice exactly through the centre of the Earth, then the circle formed would be a small circle. (Note that the two pieces of sphere would be different sizes). Examples of small circles on the Earth's surface are lines of latitude *except* for the equator.
Meridians of Longitude:	These are great circles that pass across both the north and south poles. Meridian lines (or really arcs) run north/south. They appear to be parallel on a flat map of the Earth, but in real 3D spherical Earth, they are not parallel, but all curve in towards each other to intersect at the north and south poles.
Parallels of Latitude:	These lines really are parallel (in both 2D and 3D space). It's a bit like a set of different sized circular discs piled on top of each other. The largest disc (or latitude circle) is the equator (which is a great circle). All of the other latitude circles (called parallels of latitude) are parallel to the equator and make small circles across the globe. Parallels of latitude run East/West.
Prime Meridian:	This is the meridian line that passes through Greenwich (near London). It is set as the reference point for all of the other meridians. Meridians of longitude are therefore considered either east or west of the Prime Meridian (ie. east or west of Greenwich), with 180°E being the same as 180°W.
Equator:	The latitude line that runs around the Earth exactly half way between the north and south poles. It forms a great circle that divides the Earth into the northern hemisphere and southern hemisphere.
Latitude:	The latitude of a place on Earth is the angle that would be made between lines drawn from that point, to the centre of the Earth and back out to a point on the equator that is directly north or south of the original point. If the point is north of the equator, then the latitude is specified as a north value and if it is south of the equator then it is specified as a south value.
Longitude:	The longitude of a place on Earth is the angle would be made between lines drawn from that point, to the centre of the Earth and back out to a point on the prime meridian that is directly east or west of the original point. If the point is east of the prime meridian, then the longitude is specified as an East value and if it is west of the prime meridian then it is specified as a West value.
Lat/Long Coordinates:	(Latitude/Longitude Coordinates). A point on the surface of the Earth can be specified by a combination of its latitude angle (N or S) followed by its longitude angle (E or W). For example, Melbourne Australia has Lat/Long coordinates of 38S/145E.

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