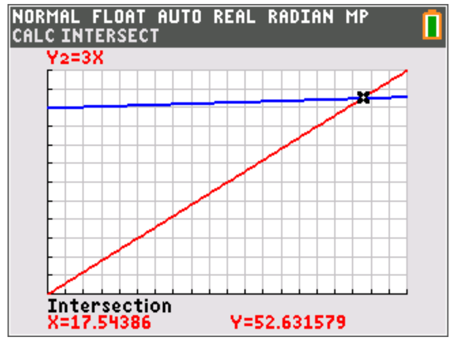
Going into Business



About the Lesson

In this activity, students will work through a scenario of a business venture involving washing dogs. They will translate fixed and variable costs to a cost function and make a decision about how much to charge to wash per dog. When the break-even point is found, students relate it back to the original scenario.
As a result, students will use systems of equations to:

- Find the break-even point (intersection of two lines).
- Interpret cost, income, and profit.

Vocabulary

- system of equations

Teacher Preparation and Notes

- Before beginning this activity, it would be helpful for students to know how to enter equations to graph and set up a table of values.
- Students must be able to solve a linear system by substitution.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

***** *with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

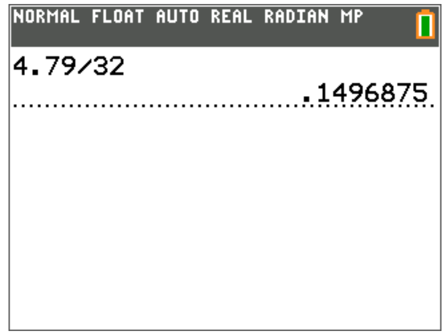
Lesson Files:

- Going_Into_Business_Student.pdf
- Going_Into_Business_Student.doc

Problem 1 – Setting Up the Problem

The first concept introduced here is the idea of a fixed cost ($50) with the purchase of supplies. You may wish to discuss start-up costs for a business. Next, the idea of a unit price is introduced. Help the students to divide the total cost of the bottle $4.79 by the number of washes (approximately 32) per bottle.

Students need to find expressions that represent the unit cost and the cost equation.



1. Find the per-dog cost for the shampoo. Describe how you found the amount.

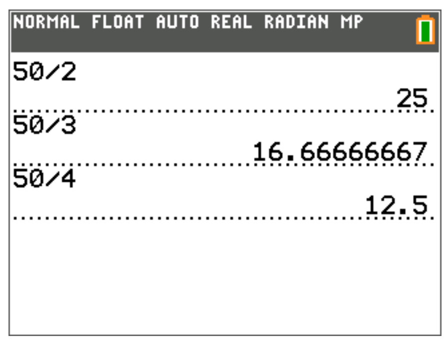
Sample Answer: about $0.15 per dog. I divided the cost for the shampoo ($4.79) by the number of washes (32).

2. Determine the cost equation for washing dogs. Don't forget that you spent $50 on supplies.

Answer: $C(x) = 0.15x + 50$

Question 3 provides an opportunity to discuss what might go into the decision of how much to charge for a service. The students could take the $50 fixed costs and divide it by several different prices to determine an approximate break-even point.

For example, if you charge $2 per dog wash, it will take about 25 dogs, whereas charging $5 per wash will speed up the profits. If the students think that a parent would give a "reasonable" figure, they might agree with the mom's offer. Have students explain why they think the price is too little or too much if they disagree with $3.

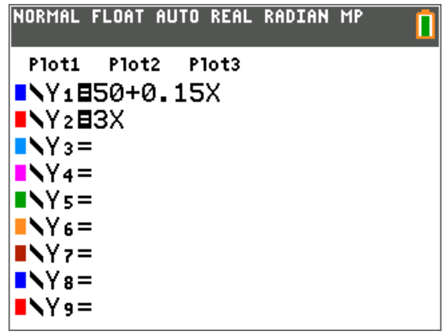


3. Your mom offers to pay $3 for your dog to be washed. Explain below why you think your mom's price is reasonable, too little, or too much. Remember, your goal is to make a profit.

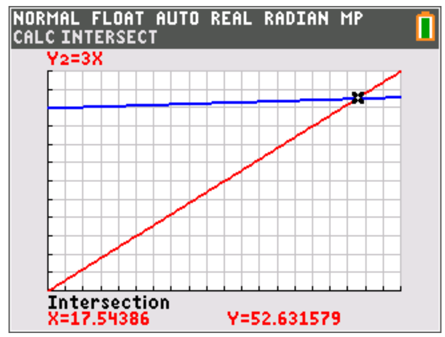
Sample Answer: Seems reasonable because the cost of the shampoo per dog is very low.

Problem 2 – Finding the Break-Even Point

Question 4 asks students to graph their revenue and costs equations. Discuss with students what each equation represents. **Y1=50+0.15X** is the cost of supplies per dog and **Y2=3X** is the money charged per wash. The settings Xmin=0, Xmax=20, Xscl=1, Ymin=0, Ymax=60, and Yscl=5 will provide a reasonable window.



Next, students find the intersection point of the two equations. To do this, press [2nd] [calc] and select **Intersect**. They will press [enter] to select the 1st equation, the 2nd equation, and a guess.



It is important to discuss the meaning of the ordered pair (17.5, 52.6). Be sure that students understand that 17.5 cannot be a number of dogs washed and that the money collected and the cost are both approximately $52.60. This is also an opportunity for students to calculate by hand washing 17 dogs and 18 dogs and then compare the answers.

Students should solve the system by substitution. This helps the student to see how the ordered pair is found, and that it represents the solution to the linear system of constraints.

4. Find the intersection point of the two lines. (Press **y/** and select **Intersect**.) Record your answer below.

Answer: (17.5, 52.6)

5. Interpret the point of intersection in terms of your business of washing dogs by explaining the meaning of the ordered pair (both *x*- and *y*-values).

Answer: The intersection point means that 17.5 dogs have to be washed to earn $52.60.

6. Solve the problem algebraically by setting the equations equal to each other. (substitution)

Answer:

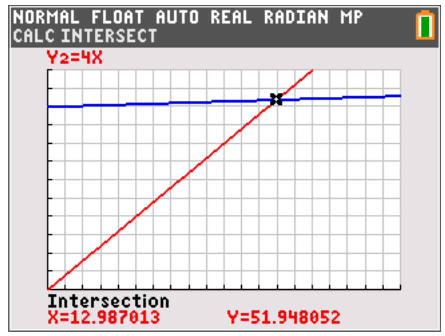
$$3x = 0.15x + 50 \qquad y = 3 \times 17.54$$
$$2.85x = 50 \qquad y = 52.63$$
$$x = 17.54$$

7. Will you make a profit before you have to buy more shampoo? (Remember that you can wash 32 dogs with one bottle of shampoo.)

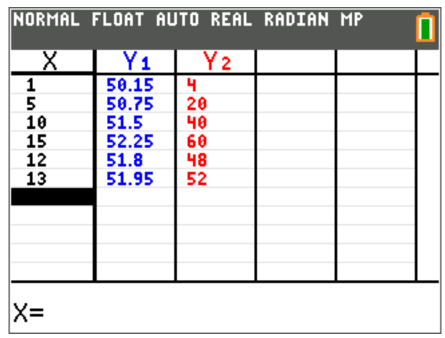
Answer: Yes, a profit will be made before more shampoo needs to be bought because any washes above 17.5 will earn profit and 17.5 is less than 32.

Problem 3 – Really Thinking About the Business Now

Students are now asked to solve the algebraic system with the altered price of $4, or **Y2=4X**. This will highlight the different solutions to the break-even point based on the price charged per dog wash.



The function table provides a different representation for the students to see how their costs and money change depending on how many dogs are washed. Students are able to calculate several different values in the table because the functions are already placed in the Y= screen.


NORMAL FLOAT AUTO REAL RADIAN MP

X	Y1	Y2
1	50.15	4
5	50.75	20
10	51.5	40
15	52.25	60
12	51.8	48
13	51.95	52

X=

8. Change the equation **Y2** which represents the amount you charge, and change it to a higher price of $4. Find the new intersection point. How does the ordered pair change as you increased the price?

Answer: The intersection point decreases.

9. What is the new system of equations? Solve the system using the $4 charge.

Answer:

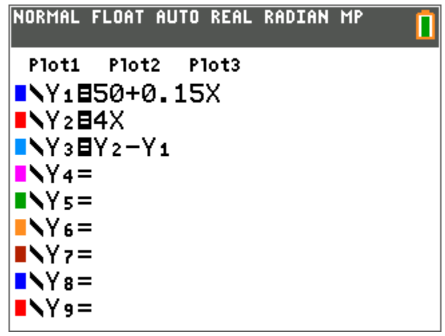
$$4x = 0.15x + 50 \qquad y = 4 \cdot 12.99$$
$$3.85x = 50 \qquad y = 51.96$$
$$x = 12.99$$

10. Calculate your profit by finding the difference in the two values. Show your work here: 13 dog washes, 14 dog washes, 15 dog washes, and 20 dog washes.

<u>**Answer:**</u> When charging $4 per wash, 13 washes: 52 – 51.95 = $0.05;
14 washes: 56 – 52.1 = $3.90; 15 washes: 60 – 52.25 = $7.75; 20 washes: 80 – 53 = $27

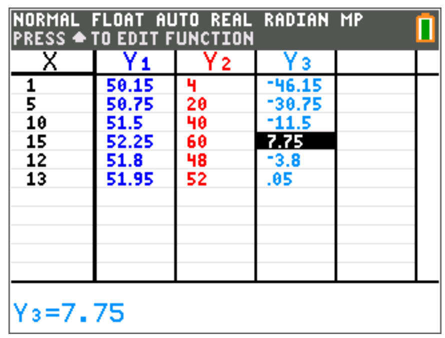
Problem 4 – Calculating the Profit

Students can use Y3 to calculate profit more easily. They need to set Y3 equal to **Y2–Y1** (income – cost). To enter Y1 and Y2, press vars, arrow to the Y-VARS menu and select **Function**. Then choose Y1 or Y2.

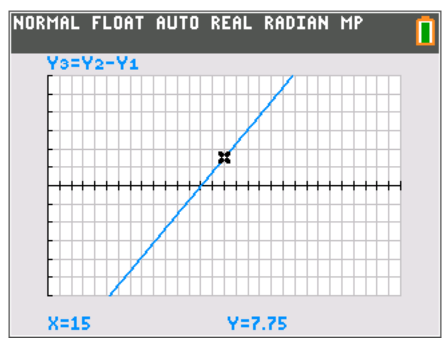


Then, students are to view the table again (2nd [table]). Discuss with students what the negative numbers mean and what the positive numbers represent. How would they find the break-even point in this representation?

Next, have students graph only Y3. Turn functions Y1 and Y2 off by placing the cursor on the equals sign and pressing enter to remove the highlight. Set an appropriate window. (one option: **Xmin** = 0, **Xmax** = 30, **Xscl** = 1, **Ymin** = –30, **Ymax** = 30, **Yscl** = 5)



Use the trace feature to explore different scenarios. For example, press trace 1 0 enter and the cursor will jump to (10, –11.5). Press 1 5 enter and the cursor will jump to (15, 7.75). Have students discuss what these values represent.



The extension questions on the worksheet allow students to investigate the problem further.

11. Enter **Y2–Y1** for **Y3** and view the table to confirm your answers from Question 10. Then calculate some additional values for numbers of dog washes, and see how much profit (or loss) you would have as a result.

Sample Answer: 1 wash is a loss of $46.15. 10 washes is a loss of $11.50. 13 washes is a profit of $0.05.

12. Graph your Profit (**Y3**). Describe the graph.

Answer: The graph is a line starting below the horizontal axis and ending above the horizontal axis.

13. What does it mean to have points on the graph that lie below the horizontal axis (y = 0)? What does the point on the graph on the horizontal axis represent? Explain what you know about points that lie on the graph above the horizontal axis.

Answer: The points below the x-axis represent not enough dogs washed in order to produce a profit. This point on the x-axis is the break-even point. The points above the x-axis are where you will begin to make a profit.

Extension/Homework

14. Explain what the result would be if you began to charge $3 for small dogs, $4 for medium, and $5 for larger dogs. How would this plan affect your profits?

Sample Answer: If, on the average, the student business owner still maintains the $4 charge, the profits will be about the same.

15. What if you hired your younger sibling (or a neighbor or a friend) to help you with the bigger dogs and the laundry? How would you determine how much to pay him or her?

Sample Answer: Fixed costs would rise, making it harder to make a profit. Perhaps a per-dog fee could be paid to the hired worker.

16. Describe how the price of the shampoo affects your equation and your profits. What if the shampoo price increases? What if it goes on sale?

Answer: If the shampoo price increases, profits will decrease. If the shampoo price decreases, profits will increase.

17. Write an algebraic equation for the profit function used when you entered **Y_2–Y_1** and the table filled itself with values of **Y_3** for you.

Answer: When charging $4 per wash, $P(x) = 2.85x – 50$.