The Movie Contract

Teacher Notes and Answers

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TI-Nspire Investigation Student 90 min

Introduction

Stella Rosengren is one of the hottest movie stars of the century. One major studio, 21st Century Dingo, was so anxious to get her to sign a film contract with them that they offered her a choice of three salary options. This exploration examines the effects of the different payment options on her final payment for the film.

Part 1: Payment schemes

Stella is offered three payment schemes for working on this movie.

- *Scheme A*: A flat rate of \$100 000 per day for as many days as the movie is being shot. That is, \$100 000 for a one-day contract, \$200 000 for a two-day contract, and so on.
- *Scheme B*: \$20 for the first day of work, but overall earnings double for each additional day of full work. That is, \$20 for a one-day contract, \$40 for a two-day contract, and so on.
- *Scheme C*: Two cents for the first day of work, but overall earnings triple for each additional day of work. That is, \$0.02 for a one-day contract, \$0.06 for a two-day contract, and so on.

Stella, although a brilliant actor and mathematician in her own right, prefers to leave financial matters to you, her agent. Your job is to give her advice about which offer to accept. First you will need to know how Stella's earnings under each of the schemes depend on number of days (x) Stella works.

Begin by examining the value of scheme A.

Question 1.

Copy and complete the following information to devise a rule for Stella's overall earnings under Scheme A [$EA(x)$] within the first week of work (7 days).

$EA(1)$ = 100 000 × 1 = \$ 100 000
$EA(2)$ = 100 000 × 2 = \$ 200 000
$EA(3)$ = 100 000 × 3 = \$ 300 000
$EA(4)$ = 100 000 × 4 = \$ 400 000
$EA(5)$ = 100 000 × 5 = \$ 500 000
$EA(6)$ = 100 000 × 6 = \$ 600 000
$EA(7)$ = 100 000 × 7 = \$ 700 000

Author: D. Tynan

Question 2.

How much would Stella earn if she worked for 10 days using scheme A?

$EA(10) = 10 \times \$100\,000 = \$1\,000\,000$

Question 3.

Explain why the rule for Stella's overall earnings using Scheme A after x days is $EA(x) = 100\,000x$.

Scheme A earnings can be calculated by multiplying the number of shooting days by $100 000 per day.

Question 4.

Scheme A is an example of a linear relationship. What does this mean?

It is a linear relationship of the form $y = mx + c$ with $m = 100000$ and $c = 0$.

Now look at scheme B, in which the amount earned by Stella doubles each day.

Question 5.

Copy and complete the following information to devise a rule for Stella's overall earnings under scheme B [$EB(x)$] within the first week of work (7 days)

$EB(1) = 20 \times 1 = \$20$

$EB(2) = 20 \times 2 = 20 \times 2^1 = \$40$

$EB(3) = 20 \times 2 \times 2 = 20 \times 2^2 = \$80$

$EB(4) = 20 \times 2 \times 2 \times 2 = 20 \times 2^3 = \$160$

$EB(5) = 20 \times 2 \times 2 \times 2 \times 2 = 20 \times\ 20 \times 2^4 = \$320$

$EB(6) = 20 \times 2 \times 2 \times 2 \times 2 \times 2 = 20 \times 2^5 = \$640$

$EB(7) = 20 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 20 \times 2^6 = \$1280$

Question 6.

How much would Stella earn if she worked for 10 days using scheme B?

$EB(10) = 20 \times 2^9 = \$10240$

Question 7.

Use the above information to find a rule for calculating Stella's overall earnings $EB(x)$ using scheme B after x days.

$EB(x) = 20 \times 2^{x-1}$

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Using a similar method, we will devise a rule for Stella's overall earnings under scheme C [EC(x)], in which the amount earnt triples each day.

Question 8.

Copy and complete the following information to devise a rule for Stella's overall earnings under scheme C [*EC(x)*] within the first week of work (7 days).

$EC(1) = 0.02 \times 1 = \$0.02$

$EC(2) = 0.02 \times 3 = 0.02 \times 3^1 = \$0.06$

$EC(3) = 0.02 \times 3 \times 3 = 0.02 \times 3^2 = \$0.18$

$EC(4) = 0.02 \times 3 \times 3 \times 3 = 0.02 \times 3^3 = \$0.54$

$EC(5) = 0.02 \times 3 \times 3 \times 3 \times 3 = 0.02 \times 3^4 = \$1.62$

$EC(6) = 0.02 \times 3 \times 3 \times 3 \times 3 \times 3 = 0.02 \times 3^5 = \$4.86$

$EC(7) = 0.02 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 0.02 \times 3^6 = \$14.58$

Question 9.

How much would Stella earn if she worked for 10 days using scheme C?

$EC(10) = 0.02 \times 3^9 = \$393.66$

Question 10.

Use the above information to find a rule for calculating Stella's overall earnings *EC(x)* using scheme C after *x* days.

$EC(x) = 0.02 \times 3^{x-1}$

Question 11.

Schemes B and C are examples of exponential relationships. What does this mean?

An exponential relationship is characterised by having a constant ratio between successive values, so in scheme B the ratio of earnings from one day to the previous day is 2, and in scheme C the ratio of earnings from one day to the previous day is 3.

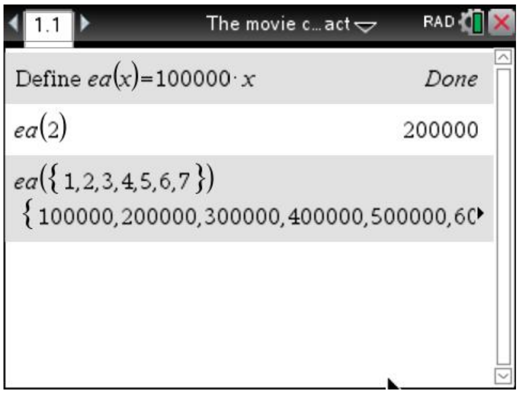
Author: D. Tynan

Part 2: Comparing the schemes numerically

The producer at 21st Century Dingo estimates that shooting the movie will take between 14 and 18 days. The TI-Nspire CAS can be used to help compare the value of each of the schemes in 2 weeks of shooting.

On the TI-Nspire CAS

- Press **HOME-1** to create a new document, and then press **1** to add a **Calculator** page
- Press **MENU-1** and then press **1** to select **Define** and then type the following to complete the rule:
  Define ea(x) = 100000x
- To use the rule to calculate the amount earned under scheme A after two days, type **ea(2)**
- To use the rule to calculate the amount earned under scheme A after each day in the first week, type **ea({1,2,3,4,5,6,7})**



Question 12.

Use your TI-Nspire CAS and the procedure outlined above to define a rule for the earnings under scheme B and scheme C. (Define rules for **eb(x)** and **ec(x)**.)

eb(x) = 20*2^(x-1) **ec(x) = 0.02*3^(x-1)**

Question 13.

Check your answers by using your rules to calculate the amount earned under scheme A, B and C for each day in the first week of shooting. (i.e. check the calculator amounts with those found previously in your answers to question 5 and 8).

Question 14.

Complete the following calculations to find which scheme would earn Stella the most money if shooting lasted 14 days.

a) *EA*(14) = $1 400 000

b) *EB*(14) = $163 840

c) *EC*(14) = $31 886.50

Question 15.

If the shooting lasts 14 days, Stella can earn most money by choosing which scheme?

If the shooting lasts 14 days, Stella can earn most money by choosing scheme A.

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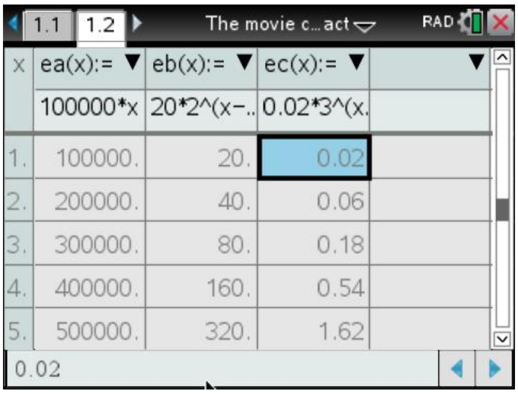
To explore and compare the value of each scheme over the days of shooting, you can use the table feature. To do this on the TI-Nspire CAS

- Press **HOME** and add a **Lists and Spreadsheet** page.
- Press **MENU-5** to select **Table**, and then press **1** to select **Switch to Table**.
- Click at the top of the first column and select **ea** to make a table of the values of *ea(x)* for various *x* values.
- Click at the top of the next column, and select **eb** to make a table of the values of *eb(x)* for various *x* values.
- Tab right to move to the next column, and select **ec** to make a table of the values of *ec(x)* for various *x* values.



Question 16.

Scroll down through the table to 10 days. The table shows that under scheme A, Stella would earn '1.E6' dollars. What does this mean?

The expression '1.E6' means 1 × 106 = 1 million.

Question 17.

Use your table to answer the following questions for 14 to 18 days of shooting.

a) For which number of days is scheme A the most lucrative?

Scheme A is most lucrative for 14–17 days.

b) For which number of days is scheme B the most lucrative?

Scheme B is most lucrative for 18 days.

c) For which number of days is scheme C the most lucrative?

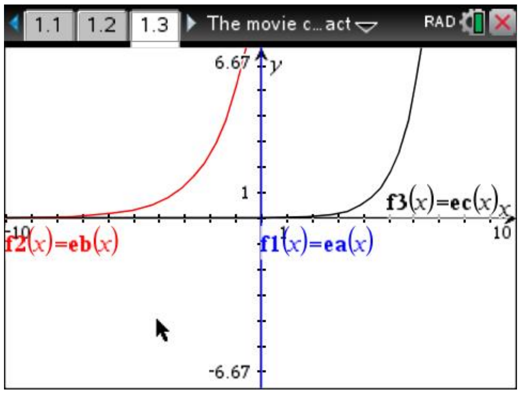
Scheme C is not the most lucrative at any time in the first 18 days (but it's close on the 18th day!).

Author: D. Tynan

Part 3: Comparing the schemes graphically

Now we will compare the earnings using graphs.

To do this using the TI-Nspire CAS



- Press **HOME** and add a **Graphs** page.
- Enter the rule for each scheme as follows (note that **CTRL-G** toggles between showing and hiding the rules)
  f1(x) = ea(x)
  f2(x) = eb(x)
  f3(x) = ec(x)
- The graph screen should appear as at right, using the standard window [–10,10] by [–6.67,6.67].

Question 18.

Is this a suitable window in which to view the graphs? Explain why/why not.

No, since we are interested in the earnings in 14–18 days, and the table shows that the earnings range between approximately 30 000 and about 2 700 000 over that time, depending on the scheme used.

Question 19.

Using the information found earlier, identify suitable minimum and maximum values (rounded) for the number of days (x) and the earnings (y) for between 14 and 18 days.

a) XMin = 14

b) XMax = 18

c) YMin = 30 000

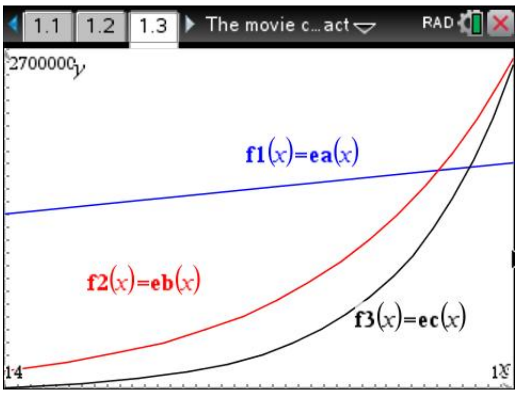
d) YMax = 2 700 000

Question 20.

Use your answer to the previous question to change the viewing window. [To modify the window settings, press **MENU-4** to select **Window/Zoom** and then **1** to select **Window Settings**].

Question 21.

Once you have located the graphs in a suitable viewing window to view earnings between 14 and 18 days, draw a sketch of the three graphs. Label each graph clearly, and the window boundaries.



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Question 22.

Compare the graphs for the earnings under each scheme. How does each graph explain how the earnings for each scheme change over the course of the 18 days?

a) Graph of scheme A

Scheme A earnings increase at a constant rate of $100 000 per day over the 18 days

b) Graph of scheme B

Scheme B earnings don't grow by as much early in the first two weeks, but appear to increase rapidly in the last week.

c) Graph of scheme C

Scheme C earnings don't grow by as much early in the first two weeks, but appear to increase rapidly in the last week, and overtake scheme B earnings after the 18th day (the 19th day).

Things go terribly wrong! As a result of a freak storm, shooting of the movie is delayed and Stella ends up working for a total of 23 days. In a moment of weakness (and clearly without checking!), the producer agrees to pay Stella under the terms of the original contract. This proves to be quite lucrative for Stella.

Question 23.

Explain why this turn of events favours Stella if she chose scheme C. How much money will she earn in this case?

Each day, scheme C triples the earnings, and considering its value after 18 days was $2.58 million, this means that its value after 23 days is approximately $627 621 192.18!

Question 24.

Explain what you have learnt from this task about the behaviour of exponential functions.

Exponential functions can change values very quickly as the values get large. The effect of doubling or tripling becomes more significant as the difference between successive values increases.

Author: D. Tynan

Teacher notes

- This task is designed to compare exponential function behaviour relative to linear function behaviour. It uses the context of payment schemes to look at how simple rules for constant change and constant multiplier impact the payments from each scheme over time. It has parallels to the famous chessboard problem (pieces of gold as payment on a chessboard).
- Students who have completed a unit on linear functions and are beginning a unit on non-linear or exponential functions would benefit from such an exploration.
- The first part of the task asks students to formulate each rule, rather than just giving them the payment rules. Some may need guidance with this, depending on their exposure to exponential function rules.
- There is a deliberate placement of the comparison of the function values (payments) via a table in the first place. This gives students a sense of the differences in the type and rate of change of each payment scheme. It also gives students a feel for what viewing window would be needed in Part 3 to get a clear picture of what is happening in the first 18 days.
- The skill of setting the viewing window values is not easy for students. Take the time to model a process for thinking carefully about what each of the parameters mean, and what the context of the problem reveals about the relevant domain and range of the rules.
- Consider trying zooming out and zooming in to locate an appropriate viewing window. Ask students to compare the efficiency of that method against directly entering sensible window values.
- Encourage the use of the terms 'constant multiplier' or 'constant ratio' to describe the changes in the payments between successive days for schemes B and C.
- Ask students whether the graph should really be drawn for positive real or positive integer values of the variable. The ease of viewing a graph is one argument for connecting the points, although the context requires that payments are based on whole days only.
- In the final part, consider showing students (or asking them to show) a window in which graphs for 0–23 days are shown. In this window, the graph for scheme A becomes almost invisible due to the relative values of earnings for 18–23 days of shooting.

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