Building Rules

TI-Nspire   Investigation   Student   30 min

Introduction

One of the aims of this activity it to build a rule that describes the number of blocks required to complete a particular pattern. The pattern may be seen in many tiling problems, a rule to determine the total number of pavers or tile is therefore an efficient means of calculating requirements if the number of tiles is likely to be very high. The real focus of this problem however, is to explore different methods for deriving a rule. One of the skills for which mathematicians are valued is their ability to solve problems in an efficient manner. Having only a single strategy is unlikely to make for efficiency, so this activity explores three different ways to solve the same problem.

Problem 1 – The Algebraic Approach

Open the TI-nspire document "Building Rules" and navigate to page 1.2. The slider in the bottom right corner of the page can be used to change the pattern number (S). Start with pattern number one (S=1) and record the number of blocks (in total) required to build the pattern.

Since: 'S' has been used in the slider to record the pattern number, all rules should use an x instead.

The number of new blocks being added each time is shown in yellow to help highlight the difference between consecutive patterns.
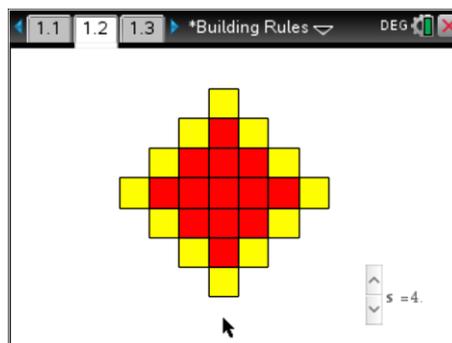
The [TAB] key can be used navigate quickly to the slider. To increase the slider amount while the slider is active (shown by black surrounding line) click up/down on the touchpad.

Question: 1.

Record the pattern number (x) and the number of blocks (bl) required to produce the entire pattern.

Pattern (x)	Blocks (bl)
1	
2	
3	
4	
5	

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Enter the results into the spreadsheet on Page 1.3.
The 'difference' columns (δ_1 and δ_2) automatically determine the successive differences between blocks (δ_1) and the differences of these differences (δ_2). These are referred to as first and second order differences.

1.3	1.4	1.5 ▶ Building Rules ⌄	RAD
◆	A pattern	B blocks	C δ_1	D δ_2
=			=δlist('bloc	=δlist('δ_1)
1	1	0	0	0
2	2	0	0	0
3	3	0	0	0
4	4	0	0	
5	5	0		

B1 | 0

With reference to difference tables, when a constant difference is determined this means two things:
- The original relationship can be expressed as a polynomial
- The degree of the polynomial is equal to the 'difference order' where the first *constant* difference occurs.

Question: 2.

After entering the 'block' results, what do the values in δ_2 say about the relationship between the pattern number (x) and the number of blocks (bl)?
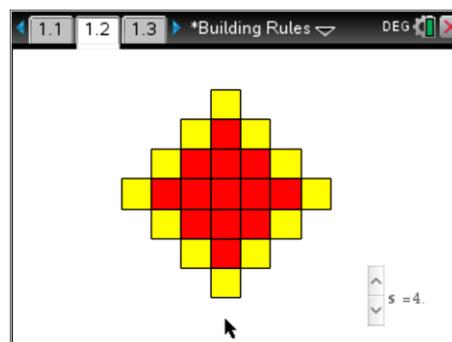
Question: 3.

Based on the results from Question 2, set up an appropriate general function that describes the relationship between the pattern number (x) and the number of blocks (bl). Using a selection of points from your data, use the solve command to determine the actual equation.

Question: 4.

Use a scatter plot in the graph application to display the pattern vs blocks data then graph the function defined in Question 3. Looking at the graph (and data), what domain restrictions should be applied to the function?

Problem 2 – CAS = Black Box

Imagine you are building the pattern by hand. The literal rule for building is: "add a square to each exposed side". The recently added squares are shown each time in yellow.
For the first stage (s = 1) no blocks are 'added', the pattern simply starts with a single block. To reach the next stage four blocks are added (s = 2).
The total number of blocks contained in the pattern is therefore the progressive **sum** of those added plus the single block required to start the pattern.

1.1	1.2	1.3 ▶ *Building Rules ⌄	DEG

s = 4.

The progressive sum is generally denoted by the Greek symbol sigma: Σ. The sum template can be found in the calculus menu in the calculator application and also in the symbols palette.

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Question: 5.

Record the pattern number and the number of **yellow** blocks required to build each stage. The first amount has been entered. (Refer to the problem description for justification of the zero)

Pattern (x)	Blocks (bl)
1	0
2	
3	
4	
5	

Question: 6.

Write a rule $ad(x)$ relating the pattern number (x) and the number of blocks **added** each time.

Question: 7.

The rule for the total number of blocks required is the progressive sum of the blocks being added and can therefore be determined using:

$$1+\sum_{x=1}^{n} ad(x)$$ --- where n represents the final shape number.

Write down the rule produced by the calculator and explain why the "1 +" is necessary.

Problem 3 – Visualising Algebra

Drawing a diagram is a very powerful problem solving tool. In this case the diagram has been drawn already, however to help focus thinking on specific aspects of the diagram a 'hint' button has been included.
With the hint off, scroll through the pattern as before.
Switch the hint on; then scroll through the pattern once again.
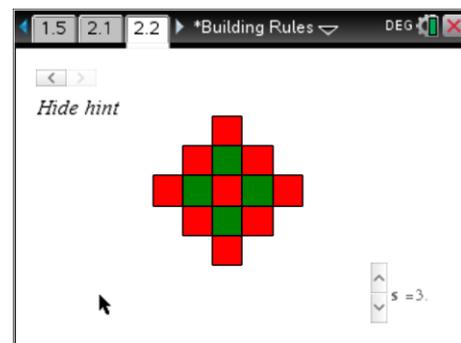
Question: 8.

Record the number of green and red blocks while the 'hint' is on. (Record the quantities only, no need to reference the colour). Use the pattern number and the quantities of red and green blocks to write a rule for the total number of blocks. Explain how you determined your rule.