

Goldbach Conjecture

Student Worksheet

7 8 9 10 11 12



TI-Nspire



Coding



Student



60 min

Conjectures

Mathematicians and scientists often develop theories, generally based on observations. They experiment with their theory and then attempt to prove it. If they are not able to prove their theory it is referred to as a conjecture. It is reported that on 7th June in 1742 that Christian Goldbach wrote a letter to Leonhard Euler in which he proposed: “Every integer greater than 2 can be written as the sum of three primes”. At the time the number one (1) was considered prime. Correspondence between the two mathematicians continued noting that “Every even integer greater than 2 can be written as the sum of two primes” ... “although I cannot prove it.”

This exchange is remarkable as mathematicians have still not managed to prove it, so it remains a conjecture and one of the many ‘unsolved mathematics problems’. In this investigation you will explore and test, many times over, whether every even number greater than 2 can be written as the sum of two prime numbers.

Question: 1.

Write each of the following even numbers as the sum of exactly two prime numbers, the first two have been done for you.

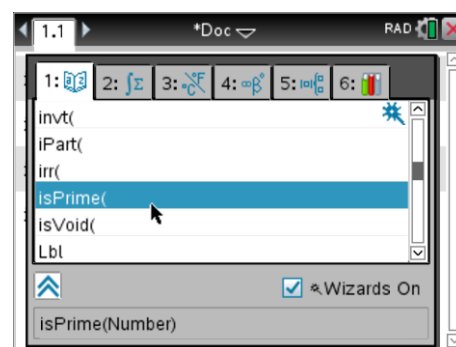
4	6	8	10	12	14	16	18
2 + 2 = 4	3 + 3 = 6						

Instructions

It would take some time just to explore the first 50 even numbers. Programs are a great way to perform such highly repetitive tasks. Fortunately the TI-Nspire calculator includes a very useful command:

‘isPrime’

The command can be typed directly or collected from the catalogue.



Question: 2.

Use the isPrime command to test the following numbers: Example: isPrime(97)

Note: The first result has been completed for you.

2	13	29	2003	2011	2013	2017	2019
True							

Question: 3.

Use your answers to the previous question to help write the following years as the sum of two prime numbers:

2014	2016	2018	2020	2022	2024	2026	2028

Coding a Solution

Insert a program application into your document by selecting:

> **Functions and Programs > Program Editor > New**

Enter the title: **Goldbach** for the name of the program and press [Enter].

Initially the page will be split into two with the calculator application on the left and the program on the right.

If you would like to see the pages separate press **Ctrl + 6**.

The first instruction in the program will be to request a number from the user.

> **I/O > Request**

Edit the command as follows:

Request "Enter an even number", n

The program will now enter a loop, searching for a pair of prime numbers that add to give 'n', the required number.

> **Control > For ... EndFor**

The FOR loop can start at 1 and end its search at n

For $i, 1, n$

The program needs to check if the current value of the counter i and the corresponding partner $n - i$ combine as a pair of prime numbers.

> **Control > If ... Then ... EndIf**

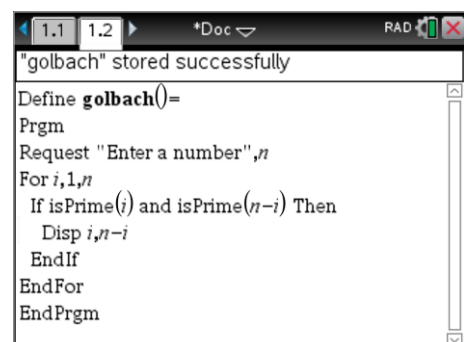
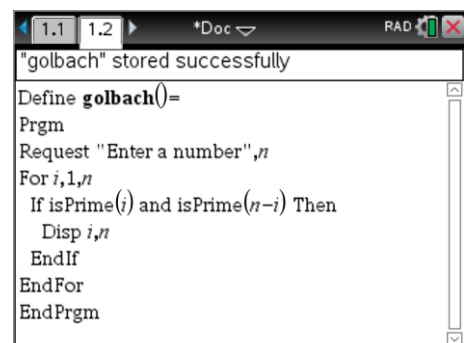
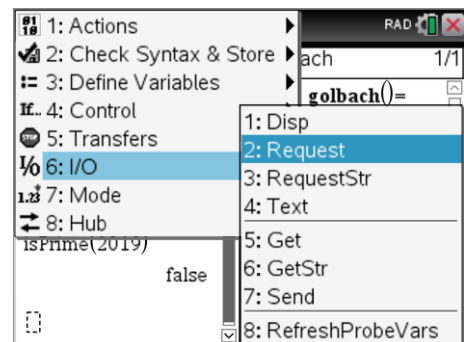
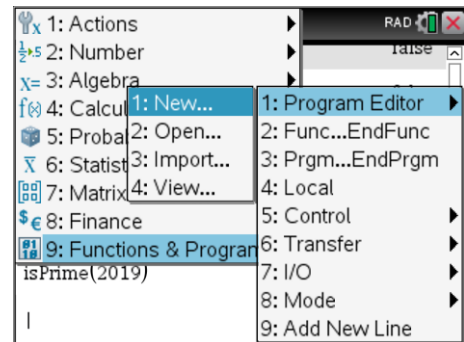
Both numbers can be checked at the same time.

If isPrime(i) and isPrime($n - i$) Then

If both of these conditions are met then the calculator will display the pair of numbers.

Disp i, n

Press **Ctrl + B** to save and compile the program.



Question: 4.

Navigate back to the calculator page. Use the [VAR] key to access and run the program. Use your program to find a pair (or pairs) of prime numbers and check your answers to Question 1.

Question: 5.

Run the program and check if there are more answers to the years selected in Question 3.

2014	2016	2018	2020	2022	2024	2026	2028

Question: 6.

Explain why changing the **FOR** loop to terminate at $n/2$ would allow the program to run in approximately half the time while still producing all the prime pairs.

Question: 7.

Alex changes his **FOR** loop to: **For** $i, 3, n/2, 2$. His program runs even faster as it only checks every second number starting with 3.

- Explain the logic behind checking every second number, starting with 3.
- Which number will be missed by Alex's program? Suggest a possible fix or adjustment.

Question: 8.

Which 2 digit even number has the most pairs of prime numbers?
Record the results of your exploration.

Question: 9.

Explain why checking all the even two digit numbers does not prove Goldbach's conjecture.

Challenge Problems

The Goldbach program returns all the prime pairs that add to a specified number. If we are trying to prove that the conjecture is true, and we are confident that the calculator is correctly finding pairs of prime numbers, it would be more efficient to simply report how many pairs have been found rather than identify the actual numbers.

Example: The number: 100 contains 6 pairs of prime numbers:
(3, 97), (11, 89), (17, 83), (29, 71), (41, 59) and (47, 53).

Challenge 1:

Change the Goldbach program to report only the quantity of prime pairs.
Include your complete program listing.

Challenge 2:

Change your Goldbach program from Challenge 1 to search all even numbers between two specified numbers returning the quantity of prime pairs found for each even number in the specified range. For example, the program output for a search from 100 to 120 would show 100,6; 102, 8; 104, 5; 106, 6; 108, 8 ...