Slope and Tangent

MATH NSPIRED

TEACHER NOTES

Math Objectives

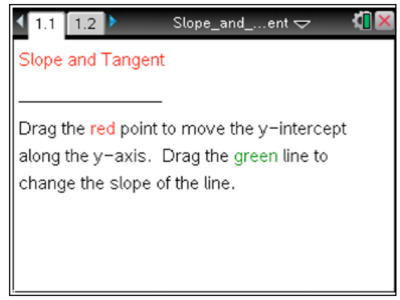
- Students will understand and be able to explain the relationship between the slope of a line and the tangent of the angle between the line and the horizontal.
- Students will be able to apply the relationship between the slope of a line and the tangent of the angle between the line and the horizontal to real life problems.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- tangent
- slope
- angle of elevation

About the Lesson

- This lesson provides opportunities for students to explore the connections between the slope of a line and the tangent of the angle between the line and the horizontal.
- Students drag a point to change the *y*-intercept of a line and drag the line to change the slope. The resulting equation of the line and the tangent of the angle between the line and the horizontal are displayed.
- Students explore the connection between the sine of the angle between a line and the horizontal and the slope of the line.
- Students apply the understanding of the slope of a line and the angle between the line and the horizontal to real life situations.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Slope_and_Tangent_Student.pdf

Slope_and_Tangent_Student.doc

TI-Nspire document

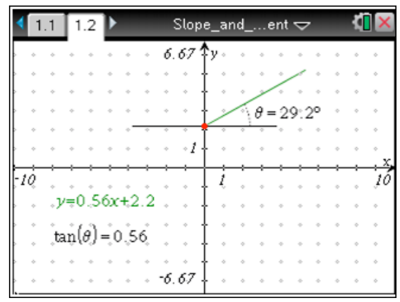
Slope_and_Tangent.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: Grab the point to drag the *y*-intercept along the y-axis. Grab the line to rotate it, changing the slope and the angle between the line and the horizontal.

Move to page 1.2.



1. Drag the *y*-intercept so that the horizontal line goes through the origin. Then drag the slanted line so that the line appears to pass through another grid point.
   a. How would you find the slope of the slanted line by hand?

      <u>Sample Answer:</u> Calculate rise over run by finding
      $$\frac{y_2 - y_1}{x_2 - x_1}$$

   b. Explain carefully how this process relates to rise over run. Where is the rise? Where is the run?

      <u>Answer:</u> The rise is the change in the *y* direction, while the run is the change in the *x* direction. The rise appears as the numerator of the ratio, while the run appears as the denominator.

   c. Now drag the *y*-intercept up and down. What happens to the slope of the slanted line as the intercept changes? Why?

      <u>Answer:</u> Changing the *y*-intercept doesn't change the slope of the slanted line. The orientation of the slanted line with respect to the horizontal remains the same, it is simply shifted up or down by changing the intercept.

2. Drag the slanted line up and down and observe the changes in the slope.
   a. The angle θ is the angle between the slanted line and a line parallel to the *x*-axis. What happens to the tangent of the angle as the slope changes?

      Answer: The tangent of the angle changes with the slope. The tangent of the angle is equal to the slope of the line.
   b. Move the slanted line back to its original position at the beginning of question 1. Explain carefully how you would find the tangent of the angle by hand.

      Answer: Create a triangle with one vertex at the *y*-intercept, one leg the horizontal line, and one leg on the slanted line. Measure the base of the triangle and the vertical leg, then find the ratio of the vertical leg to the horizontal base. This ratio is the tangent of the angle between the line and the horizontal.

   c. Why does it make sense that the tangent of the angle between the slanted line and the *x*-axis is the same as the slope of the line?

      Answer: When a triangle is built along the line, with one leg the horizontal, it is the same as using the non-intercept vertex on the line as the second point on the line. In this case, the rise of the line is the vertical leg of the triangle, while the run is the horizontal leg. Thus the tangent of the angle, the ratio of the length of the vertical leg to the length of the horizontal leg, is the same as the ratio of the rise to the run, or the slope of the line.

3. Move the slanted line so that it decreases from left to right, and the right hand side of the line is below the line parallel to the *x*-axis.
   a. What happens to the relationship between the slope and the tangent of the angle here?

      Answer: The tangent line is positive, while the slope is negative.

   b. How do you explain the difference between the two values?

      Answer: The angle is being measured from the "longer" side of the line to the horizontal, rather than as the smaller angle between the line and the horizontal.

Teacher Tip: You might want to have students investigate the periodic nature of the tangent function. For what angles is the tangent the same? For example, given an angle θ between 0 and 90 degrees, what angle between 90 and 360 degrees will have the same tangent? How do you know?

c. Write a rule relating the slope of a line to the tangent of the angle between the line and the *x*-axis, taking into account the possible difference in sign between the slope and the tangent. Explain why your rule makes sense.

Answer: As long as the angle is measured as the smallest angle between the line and the horizontal, allowing for the angle to be negative if the line is below the horizontal, then the tangent of the angle and the slope of the line will be the same.

4. What if you drew another horizontal line (parallel to the *x*-axis)? Would the relationship between the slope of the slanted line and the tangent of the angle formed with this horizontal line still hold? Explain.

Answer: Yes, the relationship would still hold. The angle would still be the same. In fact, we could construct a horizontal line from any point on the line, measure the angle between the line and that horizontal, and the tangent of that angle would equal the slope of the line.

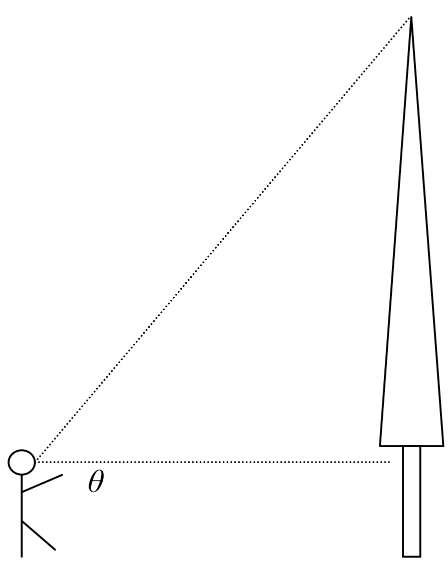
5. Suppose you did not know the tangent of the angle between the line and the *x*-axis, but you did know the sine of the angle. Could you determine the slope? Would you need any other information? Explain. You might want to calculate the sine of the angle on the line given for insight.

Answer: The sine of the angle wouldn't give you enough information, without doing some other calculations, to determine the slope of the line. The sine of an angle gives you the magnitude of the rise of the line over a distance of 1 unit *on the line*. The slope of the line is the rise over one unit *of run*. Without either finding the cosine or using the Pythagorean theorem, the sine of an angle doesn't provide enough information to determine the slope. Furthermore, the sine could be, say, positive, when the slope of the line is actually negative.

6. Suppose you were building a ramp, and you knew that the building code required that the ramp be at an angle of between 12° and 15°. How could you use this information and the work in the preceding questions to find the dimensions (i.e. length and height) of your ramp?

   Answer: We know that the tangent of the angle must be the same as the slope of the ramp. Thus the slope of the ramp must be between $\tan(12) \approx .2125$ and $\tan(15) \approx .2679$. Thus for every foot in length, the ramp must rise between .2125 and .2679 feet. So, for example, a ratio of ¼ would suffice. Then, for every 4 feet in length, the ramp could rise 1 foot. If the height between the ground and the entrance is 3 feet, the ramp will need to be 12 feet long.

7. The angle of elevation is the angle from the horizontal to an object. For example, if you draw an imaginary horizontal line at your eye level, the angle between that line and your line of sight to the top of a tree is the angle of elevation (see the picture below). If your eye level is 66 inches above the ground, and you are standing on level ground looking at a tree in your yard that is 22 feet tall and 12 feet away, what is the angle of elevation from your eye level to the top of the tree? Explain.



   Answer: The difference in height between you and the tree is 16.5 feet, and the tree is 12 feet away. Thus the slope of the line between you and the tree is $\frac{16.5}{12}$, meaning that the tangent of the angle of elevation between you and the tree is $\theta = \tan^{-1}\left(\frac{16.5}{12}\right)$, or θ is approximately 54°.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The slope of a line is the same as the tangent of the angle between the line and the horizontal.
- Knowing the sine of the angle between a line and the horizontal is not sufficient information to determine the slope of the line.
- The relationship between slope and tangent of an angle can be used to solve real life problems.