String Graphs – Part 1 + 2

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Professional Development from Texas Instruments

Student Activity

7 8 9 10 **11** 12

TI-Nspire | Investigation | Student | 45 min

Aim

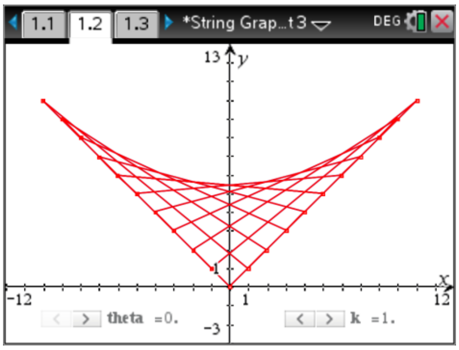
- Connect the outcomes of Advanced Strings Graphs Part 1 and Advanced Strings Part 2 using transformation matrices

Visualising the Connection

Open the TI-Nspire file String Graphs 3.

Page 1.2 contains a visual of the String Graphs produced in Activity 2. Two matrices control the location of these string patterns:

- Rotation matrix
- Dilation matrix



The angle θ (theta) is associated with the rotational matrix and can be changed using the slider. The dilation matrix dilates in both the x and y direction and can be adjusted using the k slider.

Adjust the sliders to map the lines and points from activity 2 to the lines and points from activity 1.

Question: 1.

What is the angle (measured in degrees) required to orient the points and lines from activity 2 back to those from activity 1? Justify your answer.

Question: 2.

What is the dilation factor required to map the points and lines from activity 2 back to those from activity 1? Justify your answer.

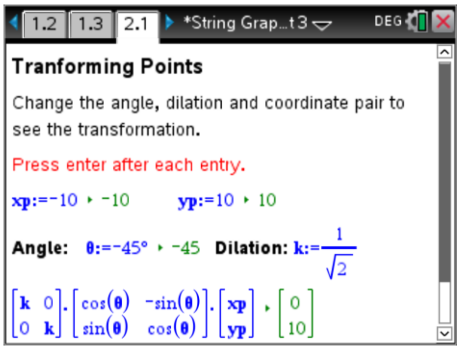
Question: 3.

Explain how the rotation and dilation connect the String Graphs activities 1 and 2.

- The dilation matrix $\begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$ represents a dilation factor k from the y axis.
- The dilation matrix $\begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$ represents a dilation factor k from the x axis.
- The rotational matrix $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ produces a rotation of θ in a counter-clockwise direction about the origin.

Author: P. Fox

Navigate to problem 2, page 1.



In this Notes application the angle, dilation and coordinates can be edited (press **Enter** after each edit). The corresponding matrix entries will automatically update.

The image opposite shows one of the original points in Activity 2 (-10, 10) transformed via a rotation in a clockwise direction (-45°) and dilated by a factor of $\frac{1}{\sqrt{2}}$ from both the y and x axis.

The resulting coordinate (0, 10) corresponds to the first point in Activity 1.

In Activity 1 points on the y axis (0, 10), (0, 9) … were connected to points along the x axis (1, 0), (2, 0) …. In Activity 2 points along the line $y = -x$, (-10, 10), (-9, 9) … were connected to points along the line $y = x$ (1, 1), (2, 2) …

Question: 4.

Use the matrix transformations on Page 2.1 to show that the points in Activity 2 can be transformed to the original points in Activity 1.

Question: 5.

The same matrix transformations on Page 2.1 can be applied to the points of intersection between consecutive lines. The first four points of intersection in Activity 2 are shown below. Determine their corresponding points in Activity 1 using the matrix transformations.

Point 1: $\left(-8, \frac{92}{11}\right)$ Point 2: $\left(-6, \frac{78}{11}\right)$

Point 3: $\left(-4, \frac{68}{11}\right)$ Point 4: $\left(-2, \frac{62}{11}\right)$

Equations

The same transformations applied to the points from Activity 1 and 2 can be applied to the lines.

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Expressions for x and y can be determined on the calculator using inverse matrix operations:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}^{-1} \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}^{-1} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

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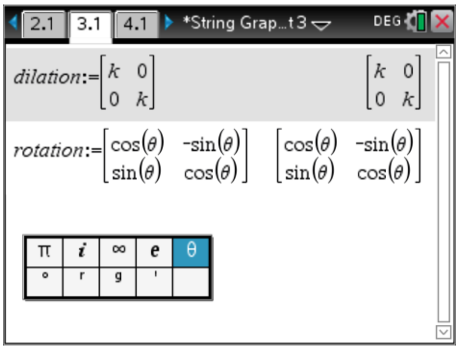
Navigate to page 3.1. The rotation and dilation matrices have already been entered so they can be copied and pasted as required.

Store -45° in angle θ

$\theta := -45^\circ$

Store $\frac{1}{\sqrt{2}}$ in the value for *k*.

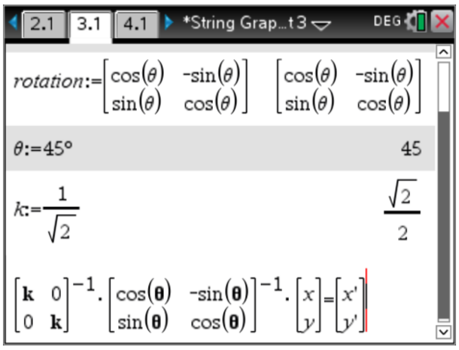


The matrix transformations can be entered naturally as they are expressed above.

Ctrl + C = Copy

Ctrl + V = Paste

An alternative method is to highlight the required expression and press [Enter] and the expression will be pasted into the active cursor position.



Question: 6.

Write expressions for x and y in terms of x' and y'.

Question: 7.

The linear equation determined in Advanced String Graphs Part 2, passing through (-10, 10) and (1, 1) is given by: $y = -\frac{9}{11}x + \frac{20}{11}$. Use your result from Question 6 to determine the linear equation passing through the points (0, 10) and (1, 0) corresponding to the first equation in Advanced String Graphs Part 1.

Question: 8.

The linear equation determined in Advanced String Graphs Part 2, passing through (-9, 9) and (2, 2) is given by: $y = -\frac{7}{11}x + \frac{36}{11}$. Use your result from Question 6 to determine the linear equation passing through the points (0, 9) and (2, 0) corresponding to the first equation in Advanced String Graphs Part 1.

Question: 9.

Navigate to page 4.1 and enter the appropriate transformations and equation, using the function notation provided: $f(x)$.

a) Check your answers to Questions 7 and 8.

b) Check the following two equations from Part 2.

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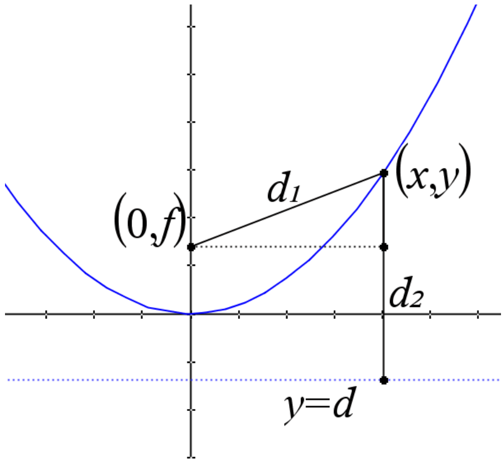
Question: 10.

The parabola passing through the points of intersection in Activity 2 was: $y=\frac{x^2}{22}+\frac{60}{11}$. Use an appropriate matrix transformation to write an equation for the equation to the curve from Activity 1.

Do not attempt to express the equation in the form y = . The equation can however be copied and pasted into the graph application on page 1.2 (Relation 1) to confirm it is correct.

Extension – Conic Sections

A parabola is defined as a set of points equidistant from a single point (focus) and a line (directrix), that is: $d_1 = d_2$ in the diagram opposite.



Question: 11.

Use the equation from Activity 2: $y=\frac{x^2}{22}+\frac{60}{11}$ to determine the location of the focus and directrix.

Question: 12.

Check your answer to the previous question using a selection of points on the curve.

Question: 13.

Use transformation matrices to determine the coordinates of the focal point and equation to the directrix for the curve from Activity 1. Use a selection of points to show that your answer is correct.

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