Air Resistance

When you solve physics problems involving free fall, often you are told to ignore air resistance and to assume the acceleration is constant. In the real world, because of air resistance, objects do not fall indefinitely with constant acceleration. One way to see this is by comparing the fall of a baseball and a sheet of paper when dropped from the same height. The baseball is still accelerating when it hits the floor. Air has a much greater effect on the motion of the paper than it does on the motion of the baseball. The paper does not accelerate for very long before air resistance reduces the acceleration so that it moves at an almost constant velocity. When an object is falling with a constant velocity, we describe it with the term *terminal velocity*, or v_T. The paper reaches terminal velocity very quickly, but on a short drop to the floor, the baseball does not.

Air resistance is sometimes referred to as a *drag force*. Experiments have been done with a variety of objects falling in air. These sometimes show that the drag force is proportional to the velocity and sometimes that the drag force is proportional to the square of the velocity. In either case, the direction of the drag force is opposite to the direction of motion. Mathematically, the drag force can be described using $F_{drag} = -bv$ or $F_{drag} = -cv^2$. The constants b and c are called the *drag coefficients* that depend on the size and shape of the object.

When falling, there are two forces acting on an object: the weight, mg, and air resistance, $-bv$ or $-cv^2$. At terminal velocity, the downward force is equal to the upward force, so $mg = -bv$ or $mg = -cv^2$, depending on whether the drag force follows the first or second relationship. In either case, since g and b or c are constants, the terminal velocity is affected by the mass of the object. Taking out the constants, this yields either

$$v_T \propto m \ \ or \ v_T^2 \propto m$$

If we plot mass versus v_T or v_T^2, we can determine which relationship is more appropriate.

In this experiment, you will measure terminal velocity as a function of mass for falling coffee filters, and use the data to choose between the two models for the drag force. Coffee filters were chosen because they are light enough to reach terminal velocity over a short distance.

OBJECTIVES

- Observe the effect of air resistance on falling coffee filters.
- Determine how air resistance and mass affect the terminal velocity of a falling object.
- Choose between two competing force models for the air resistance on falling coffee filters.
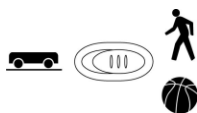
MATERIALS

TI-Nspire handheld **or**
   computer and TI-Nspire software
CBR 2 **or** Go! Motion **or**
   Motion Detector and data-collection interface
5 basket-style coffee filters

PRELIMINARY QUESTIONS

1. Hold a single coffee filter in your hand. Release it and watch it fall to the ground. Next, nest two filters and release them. Did two filters fall faster, slower, or at the same rate as one filter? What kind of mathematical relationship do you predict will exist between the velocity of fall and the number of filters?

2. If there were no air resistance, how would the rate of fall of a coffee filter compare to the rate of fall of a baseball?

3. Sketch your prediction of a graph of the velocity *vs.* time for one falling coffee filter.

4. When the filter reaches terminal velocity, what is the net force acting upon it?

PROCEDURE

1. Position the Motion Detector on the floor, pointing up, as shown in Figure 1.

2. Connect the Motion Detector to the data-collection interface. Connect the interface to the TI-Nspire handheld or computer. (If you are using a CBR 2 or Go! Motion, you do not need a data-collection interface.)

3. Set up the DataQuest Application for data collection.

   a. Choose New Experiment from the 🧪 Experiment menu.
   b. For this experiment, the default collection rate of 20 samples per second for 5 seconds will be used.
   c. Click on Graph View 🗠 to display the graph.
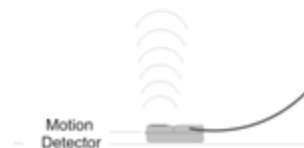   d. Select Show Graph ▶ Graph 1 from the 🗠 Graph Menu to show only the Position vs. Time graph.

   Figure 1

4. Hold a coffee filter about 1 m above the Motion Detector. Click the play 🢒 button to start data collection. After a moment, release the coffee filter so that it falls toward the motion detector on the floor.

5. Examine your position graph. At the start of the graph, there should be a region of decreasing slope (increasing velocity in the downward direction), and then the plot should become linear. If the motion of the filter was too erratic to get a smooth graph, you will need to repeat the measurement. With practice, the filter will fall almost straight down with little sideways motion. If necessary, collect the data again by simply clicking the play 🢒 button to start data collection when you are ready to release the filter. Continue to repeat this process until you get a smooth graph.

6. The linear portion of the position vs. time graph is where the filter was falling with a constant or terminal velocity (v_T). This velocity can be determined from the slope of the linear portion of the position *vs.* time graph.

   a. Select the data in the linear region of the graph.
      - **For the handheld** – move the cursor ↖ to the start of the linear region. Press and hold the center click button (🖱) until the cursor changes to ↔. Move the cursor to the end of the linear region by sliding your finger across the touchpad in the direction you want the cursor to

move. (For clickpad handhelds, use the left or right arrow keys to move the cursor.) Press esc to complete the selection.

   - **For the computer software** – click and drag the cursor across the desired region.

   b. Choose Curve Fit ► Linear from the ⬠ Analyze menu.

   c. Record the magnitude of the slope, m, as the terminal velocity in the data table. Select OK.

7. Repeat Steps 4–6 for two, three, four, and five coffee filters. Click the Store Data button ✔ before each collection with an additional filter.

8. If desired, extend to six, seven and eight filters, but be sure to use a sufficient fall distance so that you have a large enough linear section of data.

DATA TABLE

Number of filters	Terminal Velocity v_T (m/s)	(Terminal Velocity)^2 v_T^2 (m^2/s^2)
1		
2		
3		
4		
5		
6		
7		
8		

ANALYSIS

1. To help choose between the two models for the drag force, plot terminal velocity, v_T, and the square of terminal velocity, v_T^2, vs. number of filters (mass).

   a. Disconnect all sensors from your handheld or computer.

   b. Insert a new problem in your TI-Nspire document and insert the DataQuest App.
      - **For the handheld** – press doc▾ ((ctrl)(⌂) for clickpad handhelds) and choose Problem from the Insert menu. Press ⌂on and then select Ⓐ Vernier DataQuest.
      - **For the computer software** - choose Problem from the Insert menu and then choose Ⓐ Vernier DataQuest from the Insert menu.

   c. Click on Table View ▦ to view the table.

   d. Double-click on the x-column to open the column options.

   e. Change the Name to **Number of Filters**. Enter **Filters** as the Short Name and leave the units blank.

   f. Change the Display Precision to show **0 Decimal places**. Select OK.

   g. Double-click on the y-column to open the column options.

   h. Change the Name to **Terminal V**. Enter **Vt** as the short name and **m/s** as the units.

   i. Select OK and enter the data in the table.

   j. Choose New Calculated Column from the ▦ Data Menu.

   k. Enter **Terminal V^2** as the Name, **Vt^2** as the Short Name, and **(m/s)^2** as the units.

l. Enter **(Terminal V)^2** as the expression. **Note:** The term "Terminal V" must exactly match the name of the column. If you are unsure how it was entered, the available column names can be found below the Expressions entry box.

m. Select OK. Enter the column values in the Data Table.

n. Click on Graph View ⊠ to view the graph.

o. Choose Select Y-axis Columns from the ⊠ Graph menu. Select the More option and select both the Terminal V and the Terminal V^2 columns to graph. Select OK.

p. Choose Window Settings from the ⊠ Graph menu. Change the X Min and Y Min values to 0. This will scale the graph to show the origin (0,0).

q. Do a proportional curve fit on the **Terminal V** data. Choose Curve Fit from the ⊠ Analyze menu, select the **Terminal V** data, and choose Proportional. Select OK.

r. Do a proportional curve fit on the **Terminal V^2** data. Choose Curve Fit from the ⊠ Analyze menu, select the **Terminal V^2** data, and choose Proportional. Select OK.

2. During terminal velocity the drag force is equal to the weight (mg) of the filter. If the drag force is proportional to velocity, then $v_T \propto m$. Or, if the drag force is proportional to the square of velocity, then $v_T^2 \propto m$. From your graphs, which proportionality is consistent with your data; that is, which graph is closer to a straight line that *goes through the origin*?

3. From the choice of proportionalities in the previous step, which of the drag force relationships ($-bv$ or $-cv^2$) appears to model the real data better? Notice that you are choosing between two different descriptions of air resistance—one or both may not correspond to what you observed.

4. How does the time of fall relate to the weight (mg) of the coffee filters (drag force)? If one filter falls in time, t, how long would it take four filters to fall, assuming the filters are always moving at terminal velocity?

EXTENSIONS

1. Make a small parachute and use the Motion Detector to analyze the air resistance and terminal velocity as the weight suspended from the chute increases. For this extension, you may wish to hold the motion detector above the parachute when collecting data.

2. Draw a free body diagram of a falling coffee filter. There are only two forces acting on the filter. Once the terminal velocity v_T has been reached, the acceleration is zero, so the net force, $\Sigma F = ma = 0$, must also be zero

$$\sum F = -mg + bv_T = 0 \qquad \text{or} \qquad \sum F = -mg + cv_T^2 = 0$$

depending on which drag force model you use. Given this, sketch plots for the terminal velocity (y axis) as a function of filter weight for each model (x axis). (Hint: Solve for v_T first.)