Basic Trigonometric Transformations

Math Objectives

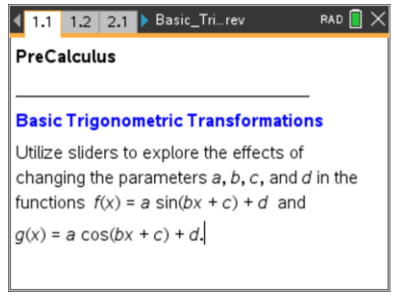
- Students will identity the amplitude, period, horizontal shift, and vertical shift in the equation $f(x) = a\sin(bx + c) + d$, and the effect that these parameters have upon the graph of the function.
- Students will discover that the horizontal shift of a sinusoidal function is determined by the quotient of two of the parameters of the function.
- Students will utilize their knowledge of the effect of the parameters of a sine function to rewrite its equation as an equation containing a cosine function.
- Students will write equations for sine and cosine functions by examining their parameters and looking at their graphs.
- Students will use technological tools to explore and deepen understanding of concepts (CCSS Mathematical Practice).
- Students will look closely to discern a pattern or structure (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- amplitude
- horizontal shift
- parameters
- period
- phase shift
- vertical shift

About the Lesson

- This lesson involves manipulating sliders to change the values of parameters in trigonometric functions and determining the effect that each change has upon the shape of the graph.
- As a result, students will:
  - Examine graphs and information about parameters of functions to write equations of sine and cosine functions.
  - Incorporate their knowledge of the effect of parameters to rewrite the equation of a sine function in terms of a cosine function.
  - Understand the effect that each of the parameters has on the graph of a function.
  - Understand how two of the parameters of a sinusoidal function effect its horizontal shift.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files: *Student Activity*
Basic_Trigonometric_Transformations_Student.pdf
Basic_Trigonometric_Transformations_Student.doc
TI-Nspire document
Basic_Trigonometric_Transformations.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

- Know how to rewrite the equation of a sine function as an equation containing a cosine function.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge and monitor students' understanding.
- Use Quick Poll to assess students' understanding and to compare their answers.

Discussion Points and Possible Answers

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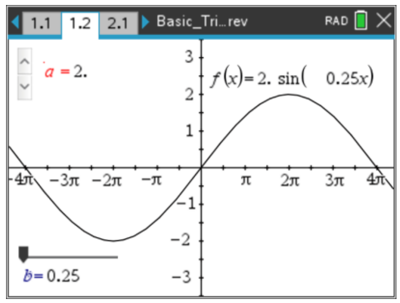

1. Drag the sliders to change the values of *a* and *b* in the function $f(x) = a \sin(bx)$.

   a. Describe how the values of *a* and *b* affect the shape of the graph.

   Answer: The sine curve is vertically stretched by a factor of |*a*|. Thus, the amplitude = |*a*|. The value of *b* affects the horizontal stretch of this function and thus changes the period of the function.

   Teacher Tip: If students do not immediately recall how the value of *b* is related to the period of the function, have them set the value of *b* to 0.25, 0.5, 1, and 2, and then have them identify the period for each (8π, 4π, 2π, and π, respectively). After some examination, students should be able to identify the relationship: period = $\frac{2\pi}{b}$.

   b. What happens to the graph if *a* is negative?

   Answer: If *a* is negative, then the curve is reflected over the *x*-axis.

c. Complete the following statement:

Answer: For $a \neq 0$ and $b > 0$, the graph of $f(x) = a \sin(bx)$ has an amplitude of $\underline{|a|}$ and a period of $\underline{\frac{2\pi}{b}}$.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

Teacher Tip: Before students drag the slider in question 2, you may want to ask them to predict what will happen by first considering a different function, $y = x^2$. Ask them how to obtain the graph of $y = x^2 + 3$ from the graph of $y = x^2$ (translate the graph up three units).

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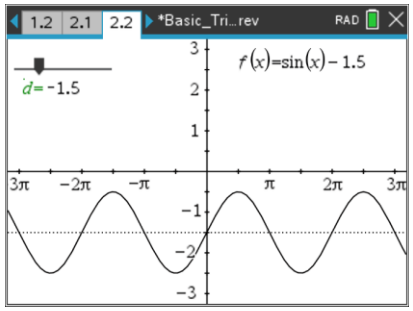

2. Drag the slider to change the value of d in the function $f(x) = \sin(x) + d$.

a. Describe how the value of d affects the shape of the graph.

Answer: The vertical shift is equal to this parameter; that is, vertical shift = d.

b. Complete the following statement:

Answer: The graph of $f(x) = \sin(x) + d$ has a vertical shift of _d_.

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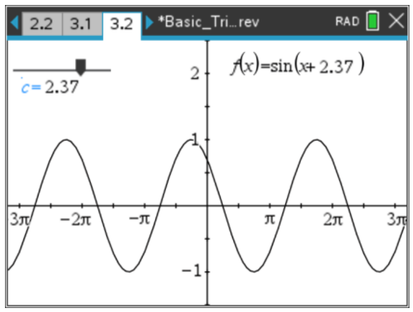

3. Drag the slider to change the value of c in the function $f(x) = \sin(x + c)$. Describe how the value of c affects the shape of the graph.

Answer: If $c > 0$, there is a horizontal shift of c units to the left. If $c < 0$, there is a horizontal shift of c units to the right. Note: This statement is only true if the coefficient of x is one.

Teacher Tip: Students might predict that a change in *c* will always result in a horizontal shift of c but how that number of units relates to *c* will not be immediately clear. This is explored in question #4.

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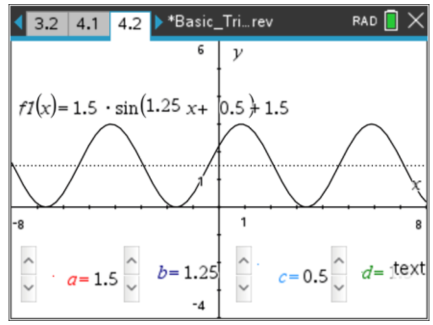

4. Drag the slider to change the values of *a*, *b*, *c*, and *d* in the function $f(x) = a\sin(bx + c) + d$.

a. Which of the four parameters have an impact on the horizontal shift of the graph?

Answer: The two parameters, *b* and *c*, affect the horizontal shift.

b. Complete the following statement:

Answer: For $a \neq 0$ and $b > 0$, the graph of $f(x) = a\sin(bx + c) + d$ has a horizontal shift of $-\frac{c}{b}$.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Teacher Tip: To establish exactly how *b* and *c* determine the horizontal shift, students can drag the sliders on Page 4.2. Encourage students to conjecture the relationship on their own, but if they need help, have them consider the horizontal shift when $b = 1$ and $c = 2$, when $b = 2$ and $c = 1$, and when $b = 2.5$ and $c = -5$. (It is –2, –0.5, and 2, respectively.) Examining these values, students should conclude that horizontal shift is $-\frac{c}{b}$.

It is easiest to identify this relationship if parameters *a* and *d* are left as initially set ($a = 1$ and $d = 0$). After the relationship is determined, dragging the sliders can verify that neither *a* nor *d* affects the horizontal shift.

5. For functions of the form $f(x) = a\sin(bx + c) + d$ or $g(x) = a\cos(bx + c) + d$, with $a \neq 0$ and $b > 0$,

Answers:

a. the amplitude is $|a|$.

b. the period is $\frac{2\pi}{b}$.

c. the horizontal shift is $-\frac{c}{b}$.

d. the vertical shift is d.

> **Teacher Tip:** Although some mathematics textbooks use the term "horizontal shift" synonymously with "phase shift", engineers and physicists make a distinction between the two terms. The phase shift enables us to calculate the fraction of a full period that the curve has been shifted to determine if two waves reinforce or cancel each other. For the sinusoidal function written in the form $f(x) = a\sin(bx + \phi) + d$ or $f(x) = a\cos(bx + \phi) + d$, ϕ is the phase shift.

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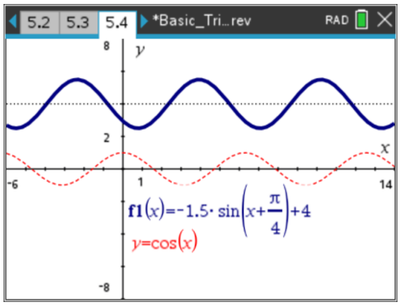

6. The function shown on this page has the equation $\mathbf{f1}(x) = -1.5\sin\left(x + \frac{\pi}{4}\right) + 4$. Write an equation for a cosine function that will have the same graph.

Sample Answers: Two possible solutions are

$y = -1.5\cos\left(x - \frac{\pi}{4}\right) + 4$ or

$y = 1.5\cos\left(x + \frac{3\pi}{4}\right) + 4$.

> **Teacher Tip:** Students should observe that the values of $|a|$, b, and d remain the same for each sine/cosine pair; the only difference occurs in the value of c. Because these functions are periodic, there are infinitely many equations that satisfy each condition. Be sure to check students' equations.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 3 at the end of this lesson.

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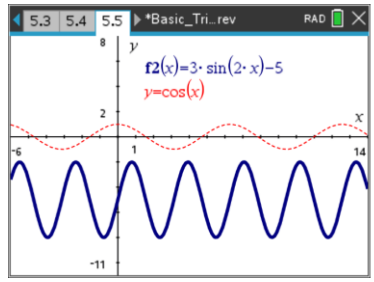


7. The function shown on this page has the equation **f2**(x) = 3 sin(2x) – 5. Write an equation for a cosine function that will have the same graph.

Sample Answers: One possible solution is $y = 3\cos\left(2x - \frac{\pi}{2}\right) - 5$. This solution can also be written as $y = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right) - 5$.

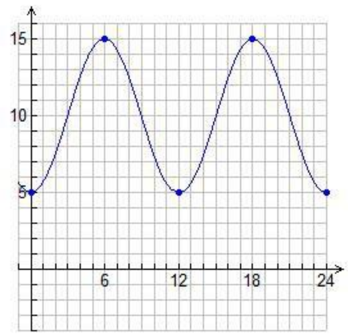
8. a. Write an equation for a sine function with an amplitude of 4, a period of 12, a horizontal shift of 2, and a vertical shift of 3.

Answer: One possible solution is **f**(x) = $4\sin\left(\frac{\pi}{6}x - \frac{\pi}{3}\right) + 3$. This solution can also be written as **f**(x) = $4\sin\left(\frac{\pi}{6}(x - 2)\right) + 3$.

b. Write an equation for a cosine function with the same parameters as the sine function in part (a).

Answer: One possible solution is **g**(x) = $4\cos\left(\frac{\pi}{6}x - \frac{5\pi}{6}\right) + 3$. This solution can also be written as **g**(x) = $4\cos\left(\frac{\pi}{6}(x - 5)\right) + 3$.

9. a. Write an equation for the sine function whose graph is shown in the figure below.



Answer: One possible solution is **f**(x) = 5 sin$\left(\frac{\pi}{6}x - \frac{\pi}{2}\right)$ + 10. This solution can also be written as **f**(x) = 5 sin$\left(\frac{\pi}{6}(x-3)\right)$ + 10.

> **Teacher Tip:** Students can check their equations by inserting a graph page on the Scratchpad. To do so, press [calculator key] on the handheld, and select the graphing application.

b. Utilize a cosine function to write an equation for the same graph.

Sample Answers: One possible solution is **f**(x) = -5 cos$\left(\frac{\pi}{6}x\right)$ + 10.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The effect that each of the parameters has on the graph of a function.
- How two of the parameters of a sinusoidal function effect its horizontal shift.
- How to rewrite the equation of a sine function as an equation containing a cosine function.
- How to write equations of sine and cosine functions by examining graphs of sinusoidal functions and information about its parameters.

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Note 1

Question 1, Name of Feature: Quick Poll

You might want to send a *Quick Poll* to *have* students check some of their answers to Questions 1 and 2.

Note 2

Question 4, Name of Feature: Quick Poll

You might want to send a *Quick Poll* to determine whether or not students comprehend using more than one transformation at a time.

Note 3

Question 6, Name of Feature: Screen Capture

You might want to use *Screen Capture* once all students have found a cosine function to match the given sine function. Scroll through the various screen captures, asking students what is common about the functions. Students should see that there is more than one correct answer. Discuss why there are multiple correct answers.