

## Claims About Two Proportions

ID: 12603

 Time required  
*45 minutes*

### Activity Overview

*In this activity, students test claims about two proportions by calculating test statistics, critical values, and P-values. They will find the P-values by finding the area under the curve of a standard normal distribution in the appropriate tail or tails. There are both one- and two-tailed tests. Once students are comfortable using the formula for the test statistic, they will learn to use the 2-Prop z test command.*

### Topic: Hypothesis Testing

- *Calculate proportions*
- *Determine test statistics for comparing two proportions*
- *Find critical values and P-values of a normal distribution*

### Teacher Preparation and Notes

- *This activity assumes students are familiar with the concepts used in hypothesis testing.*
- *For each problem, have students check that the binomial distribution can be approximated by the normal distribution by finding  $n_1\hat{p}$ ,  $n_1(1 - \hat{p})$ ,  $n_2\hat{p}$ , and  $n_2(1 - \hat{p})$ . These values can be estimated using mental math.*
- ***To download the student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter "12603" in the keyword search box.***

### Associated Materials

- *ClaimsTwoProportions\_Student.doc*

### Suggested Related Activities

*To download any activity listed, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter the number in the keyword search box.*

- *Testing Claims About Proportions (TI-84 Plus family) — 10130*
- *Difference Between Two Proportions (TI-84 Plus family) — 10081*
- *Comparing Two Means (TI-84 Plus family) — 10258*

**Problem 1 – I Deserve a Raise!**

Introduce and discuss the test statistic that is used for testing a claim about two proportions.

The scenario for the problem is given on the worksheet. Students should use  $\alpha = 0.05$  to test the claim that a greater proportion of women did not receive raises.

The null and alternative hypotheses for this right-tailed test is:  $H_0: p_1 \leq p_2$ ,  $H_a: p_1 > p_2$ .

Students are to find  $\hat{p}$ ,  $\hat{p}_1$ , and  $\hat{p}_2$ , the values they need for calculating the test statistic, z. They should store each value when calculated by choosing **[STO]** and entering a variable.

For example, in the screenshot, variables P, A, and B have been used to represent  $\hat{p}$ ,  $\hat{p}_1$ , and  $\hat{p}_2$  respectively.

Students will then find the test statistic, z, using the stored values of P, A, and B. Warn them to be careful with their placement of parenthesis. They should find that z is about 1.48648. Explain to students that they do not know if this value is, or is not, in the critical region until they know the critical value.

Students are to find the critical value by using the **invNorm** command. (Press **[2nd]** **[DISTR]** and choose **3: invNorm(**

Because this is a right-tailed test, all of the 0.05 is to the right of the critical value. The critical value is 1.64485.

Test statistics greater than this value are in the critical region and not likely to occur when the null hypothesis is true.

Ask students what their decision about the hypothesis is and why.

The test statistic is not in the critical region. There is not sufficient evidence to reject the null hypothesis, meaning we cannot say that the proportion of women who did not receive raises was less than the proportion of men who did not receive raises.

Test Statistic for Two Proportions:

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p} \cdot (1 - \hat{p}) \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{p}_1 = \frac{x_1}{n_1}, \quad \hat{p}_2 = \frac{x_2}{n_2}, \quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

```
(138+75)/(1562+1041)+P
      .0818286592
138/1562+A
      .0883482714
75/1041+B
      .0720461095
```

```
(A-B)/√(P*(1-P)*((1/1562)+(1/1041)))
      1.486481177
```

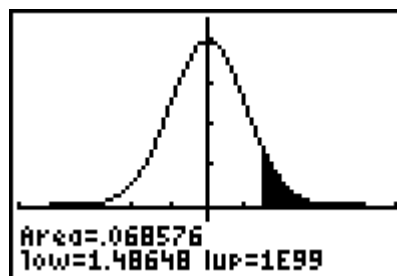
```
invNorm(.95,0,1)
      1.644853626
```

This decision can also be based on looking at the  $P$ -value. To find it, students will graph the normal distribution with a mean of 0 and a standard deviation of 1 in an appropriate window (-5 to 5 for  $x$  and -0.1 to 0.5 for  $y$ ).

The **ShadeNorm** command will draw the graph, shade the critical region, and calculate the  $P$ -value. The value 1E99 represents positive infinity. To enter E, students can press  $\boxed{2nd} \boxed{[EE]}$ .

The area is greater than 5%. The null hypothesis is only rejected when the  $P$ -value is less than alpha, 0.05 for this problem.

```
ShadeNorm(1.4864
8, 1E99)
```



### Problem 2 – Special Training

Introduce the scenario from problem 2 on the worksheet, and then have students write the null and alternative hypotheses for this two-tailed test.

$$H_0: p_1 = p_2, H_a: p_1 \neq p_2.$$

Students are to find the values of  $\hat{p}$ ,  $\hat{p}_1$ , and  $\hat{p}_2$ .

They can use the same variables to store the values as they did in the previous problem.

$$\hat{p} = 0.728, \hat{p}_1 = 0.654, \hat{p}_2 = 0.8$$

```
(51+64)/(78+80)→
P
.7278481013
51/78→A
.6538461538
64/80→B
.8
```

#### If using Mathprint OS:

Students can calculate the values using the fraction template. To do this, press  $\boxed{ALPHA} \boxed{[F1]}$  and select **n/d**. Then enter the value of the numerator, press  $\boxed{\downarrow}$  to move to the denominator. Enter the value of the denominator and press  $\boxed{ENTER}$  to evaluate.

Press  $\boxed{\rightarrow}$  to move out of the fraction. Also, parentheses are not needed around the numerator or denominator.

```
51+64
78+80 →P
.7278481013
51
78 →A
.6538461538
```

Students then find the test statistic.

$$\frac{(A-B)}{\sqrt{P(1-P)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

-2.063706533

**If using Mathprint OS:**

Students can calculate the values using the fraction template within a fraction template. To do this, press  $\boxed{\text{ALPHA}} \boxed{[F1]}$  and select **n/d**. Then enter the value of the numerator, press  $\boxed{\downarrow}$  to move to the denominator. Press  $\boxed{2\text{nd}} \boxed{[\sqrt{\quad}]}$ , the cursor will be under the radical sign. Enter **P(1-P)** (and then press  $\boxed{\text{ALPHA}} \boxed{[F1]}$  and select **n/d**. again to create a fraction within the denominator. Enter the value of the denominator. Press  $\boxed{\rightarrow}$  to move out of the fraction. Create another fraction template and close the parentheses.

$$\frac{A-B}{\sqrt{P(1-P)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

-2.063706533

Also, parentheses are not needed around the numerator.

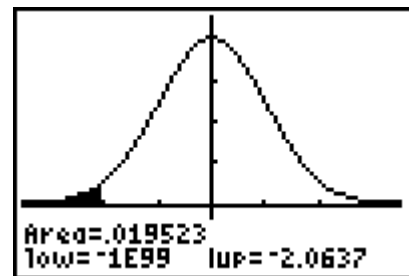
Students then find the critical values using the **invNorm** command as they did in Problem 1.

invNorm(0.025, 0, 1)  
-1.959963986  
invNorm(0.975, 0, 1)  
1.959963986

Remind students that this is a two-tailed test, so 2.5% of the area under the curve is in each tail.

Have students discuss their decision about the hypothesis. They should reject the null hypothesis because the test statistic is in a critical region. The data suggests that there may be a difference in the proportions of men and women who went to the training.

Ask students what they know must be true about the *P*-value. (It is less than 0.05.)



Then, have students calculate the *P*-value by finding the area in one tail, using the **ShadeNorm** command as before (this time using  $-1E99$ ), and doubling it.

The *P*-value is about 0.039. Discuss with students how the *P*-value can support their reasoning.

**Problem 3 – Job Satisfaction**

Introduce the in Problem 3 on the worksheet. Point out that this time; the significance level is at 10%, or 0.10.

Have students write null and alternative hypotheses for this two-tailed test.

$$H_0: p_1 = p_2, H_a: p_1 \neq p_2$$

The scenario gives each proportion, but not the number of successes, that is, the number of men and women satisfied with their jobs. Because they will not be calculating the test statistic in this problem, students need to multiply to find  $x_1$  and  $x_2$ .

$0.82 \cdot 150$	123
$0.88 \cdot 150$	132

Show students how to use the **2-Prop z Test** command to determine the  $P$ -value. They should press **STAT** and choose **2-PropZTest** from the **TESTS** menu. Tell them to fill in the problem data as shown, choose the two-tailed alternative hypothesis, highlight **Calculate** and press **ENTER**.

```
2-PropZTest
x1:123
n1:150
x2:132
n2:150
P1:P1 <P2 >P2
Calculate Draw
```

Students may choose the **Draw** option instead of calculate to see the results displayed graphically. (It will automatically use appropriate window settings.)

```
2-PropZTest
P1≠P2
z=-1.45521375
P=.1456101743
P1=.82
P2=.88
↓P=.85
```

Students will see that the  $P$ -value is about 0.146.

Have students discuss their decision about the hypothesis.

They should fail to reject the null hypothesis because the  $P$ -value is not less than 0.10. There is not enough evidence to suggest that the proportion of men satisfied with their job differs significantly from the proportion of women satisfied with their job.