Thursday Night Precalculus Series
April 4, 2024

In this AP Precalculus Live session, we will explore questions which use technology to prepare for the AP Precalculus Exam.

Looking Ahead: Tips for the AP® Precalculus Exam

About the Lesson

- This Teacher Notes guide is designed to be used in conjunction with the AP Precalculus Live session and Student Problems document that can be found on-demand:
  - https://www.youtube.com/live/UCb-endHGIQ?si=uVBT4ePTjOH8Fvyz
  - *Please note that not all problems/content from the Student Problem Sheet is covered in the video component. Student/Teacher Notes are also useful without students viewing the "Live Session" but can be enriched by that resource.*
- This session involves exploring a variety of questions that involve the use of the graphing calculator, such as:
  - Input output values and zeros of a function.
  - Regression.
  - Exponential models.
  - Logarithmic functions.
  - Piecewise defined functions.
- Students should be able to use the TI-84 to answer multiple choice and free response questions on the AP Precalculus Exam.

Class Discussion: Use these questions to help students communicate their understanding of the problem. These questions are presented in the *Live* video as well.

Materials:
Student document
- Precal_problems_04_04

Teacher document
- problems_solutions_04_04

YouTube
https://www.youtube.com/live/UCb-endHGIQ?si=uVBT4ePTjOH8Fvyz

AP Precalculus Mathematical Practices

- 1.A: Solve equations and inequalities represented analytically, with and without technology.
- 1.C: Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.

- 2.A: Identify information form graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
- 2.B: construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.

Source: AP Precalculus Course and Exam Description, The College Board

Problem 1.

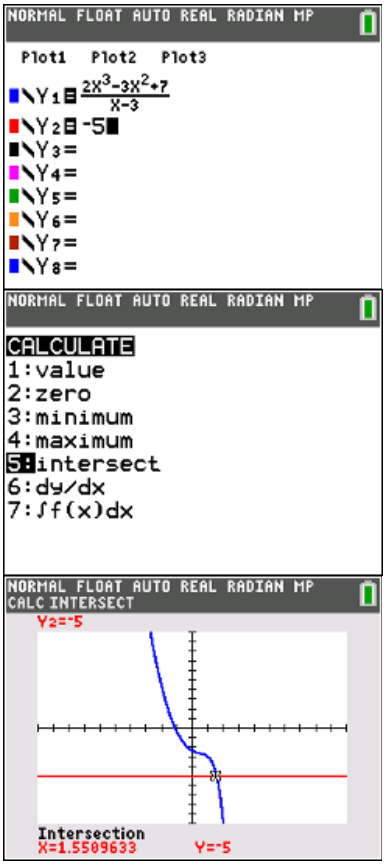
The function f is defined by $f(x)=\dfrac{2x^3-3x^2+7}{x-3}$. What input value(s) in the domain of f yields an output value of -5?

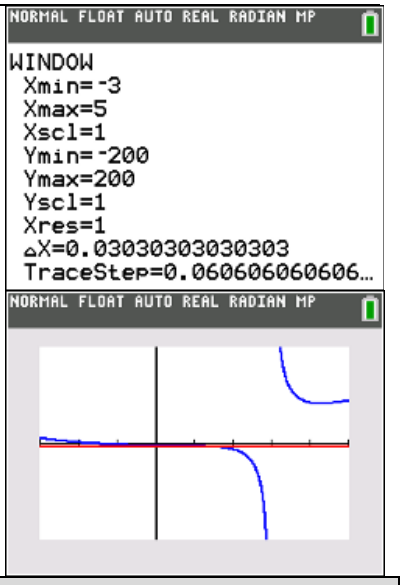
Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Technology Tip: Select alpha [X,T,θ,n] to have a fraction template. Graph using a standard window. Select 2nd [trace] to calculate an intersection point.

Technology Tip: If you need the x value of the intersection point for another calculation, go back to the calculator screen and type [X,T,θ,n] enter.





Class Discussion:

Should we be concerned about what is happening with the graph to the right of 5?

Possible Answers: Yes, if the student is using the graph of f to find the input when the output is –5, then it would be a good idea to explore the graph by changing the window.

Class Discussion:

What is going on with this graph at $x = 3$?

Possible Answers: The graph has a vertical asymptote.

Problem 2.

The table shows values for a function f at selected values of x.

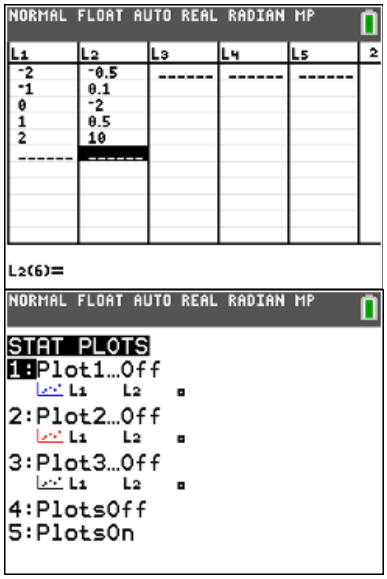
x	–2	–1	0	1	2
$f(x)$	–0.5	0.1	–2	0.5	10

A cubic regression is used to model the function f. What is the value of $f(0.5)$ predicted by the cubic regression model?

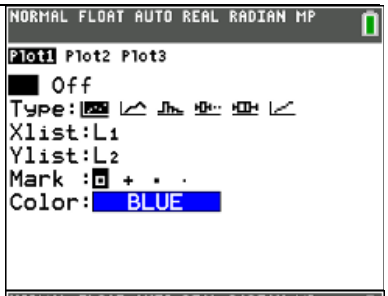
Sample Solution:

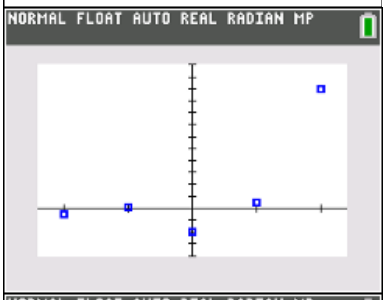
Refer to the Teacher Solutions Document for the full solution to this problem.

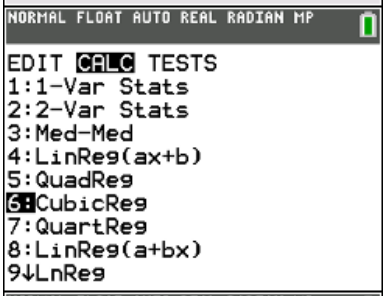
Technology Tip: Select [stat] and 1: Edit to enter the data. [L1] will represent the x-values and [L2] will represent the function values.
Go to the Stat Plot ([2nd][y=]). Use Zoom Stat.

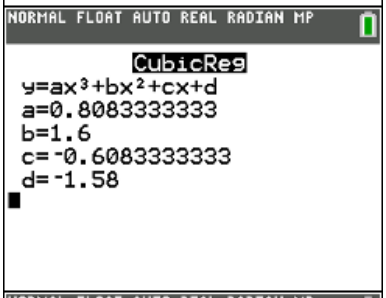


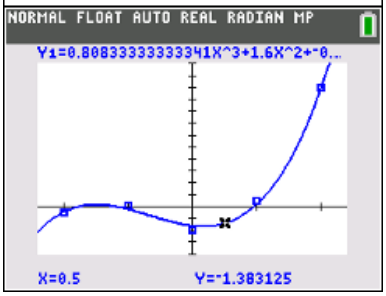
Technology Tip: Go back to the Stat menu and select Calc. Select 6: CubicReg.
Store the regression in Y1. Select alpha trace as a shortcut to select Y1.
Either the graph screen or the calculator screen may be used to find Y1(0.5).

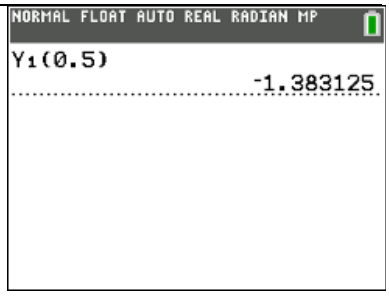












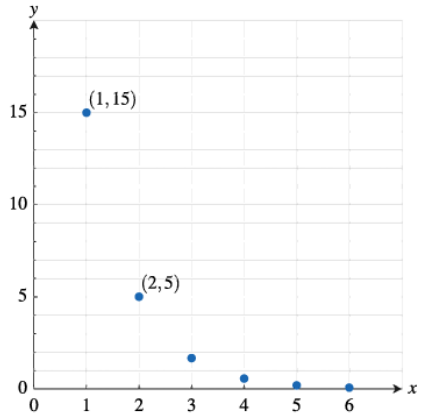
Class Discussion:

Does the model underestimate or overestimate the actual value at $x = -1$?

Possible Answers: The model predicts –0.18, but the given value is 0.1. The model predicts a value below the actual value, so it is an underestimate.

Problem 3.

A geometric sequence has the form $g_n = g_k \cdot r^{(n-k)}$. The graph of a geometric sequence, g_n, is shown in the figure.



What is the value of g_5?

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

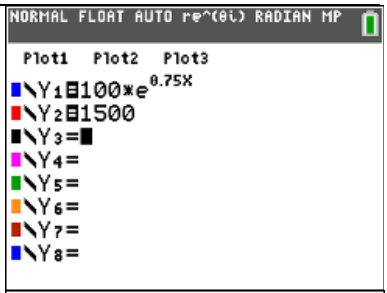
Problem 4.

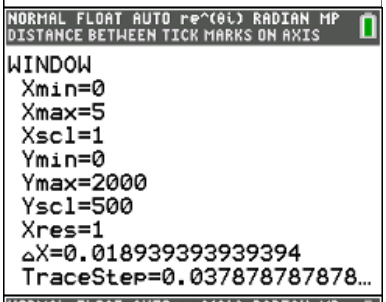
The growth of bacteria in a culture is modeled by $y = 100e^{0.75t}$, where t is measured in days. At what time t is the number of bacteria approximately 1500?

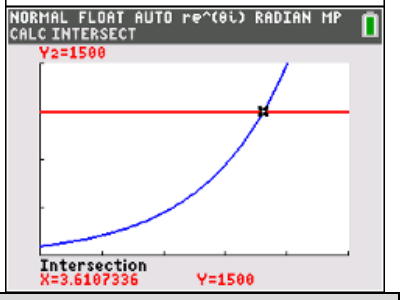
Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Technology Tip: Graph using a window shown to the right. Select 2nd trace to calculate an intersection point.







Class Discussion:

How do students know that there is only one solution?

Possible Answers: This exponential function is increasing and concave up so there is only one intersection point.

Problem 5.

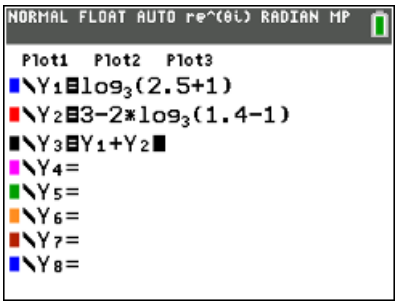
Consider the logarithmic functions f and g defined by $f(x)=\log_3(2.5x+1)$ and $g(x)=3-2\log_3(1.4x-1)$. Find a zero of the function h defined by $h(x)=f(x)+g(x)$.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

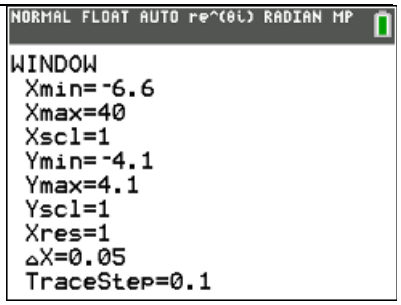
Technology Tip: Select Math > A: logBASE(to enter a logarithmic function with a base of 3. A shortcut is alpha window.

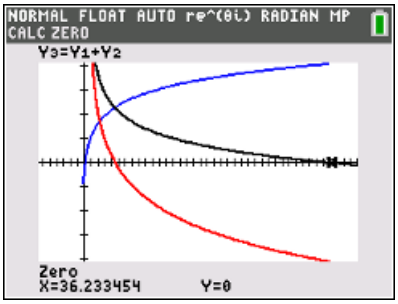
For Y3 enter Y1 + Y2. Y1 and Y2 are found using vars or alpha trace.



Technology Tip: Trace will identify which graph we are interested in using.

Graph using a window shown to the right.
Select 2nd trace to calculate a zero.





Class Discussion:

In the algebraic solution using the quadratic formula, there are two solutions. Why do we have only the one solution of 36.233?

Possible Answers: The quadratic formula solution of –0.366 is called an extraneous solution. The value of –0.366 is not in the domain of the function g.

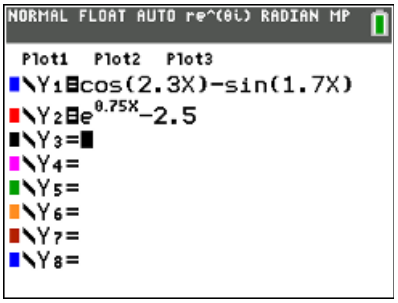
Problem 6.

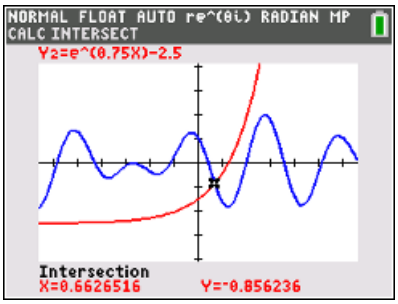
The function f is given by $f(x)=\cos(2.3x)-\sin(1.7x)$. The function g is given by $g(x)=e^{0.75x}-2.5$. Find the input value such that $f(x)=g(x)$.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Graph the functions using a Zoom Decimal window. Find the intersection point.





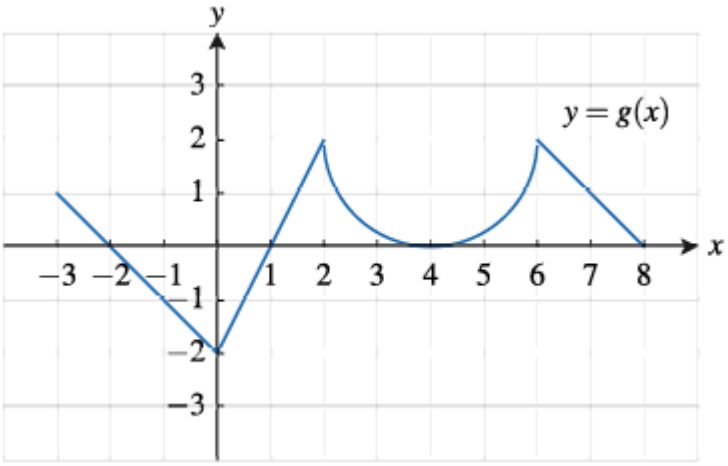
Class Discussion:

How do we use the features of an exponential function to be certain that there is only one intersection point?

Possible Answers: This exponential function g has a vertical translation down 2.5 units and is always increasing; therefore, g will only intersect the periodic function once.

Problem 7. (A) – (C)

The graph of the function g is shown in the figure and consists of three line segments and a semicircle with radius 2.



The function f is given by $f(x)=\dfrac{-3x^2+1.9x+4.5}{x^3+2x^2+1}$.

(A) (i) The function h is defined by $h(x)=(f\circ g)(x)=f(g(x))$. Find the value of $h(7)$, or indicate that it is not defined.

(ii) Find all values of x for which $g(x)=-1$, or indicate there are no such values.

(B) (i) Find all real zeros of f, or indicate there are no such values.

(ii) Determine the end behavior of f as x increases without bound. Express your answer using the mathematical notation of a limit.

(C) (i) Determine if an inverse function of g can be constructed for all values of x in the closed interval $[2,6]$.

(ii) Give a reason for your answer based on the definition of a function and the graph of g.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Technology Tip: Use the fraction template in the [y=] to define the function f.

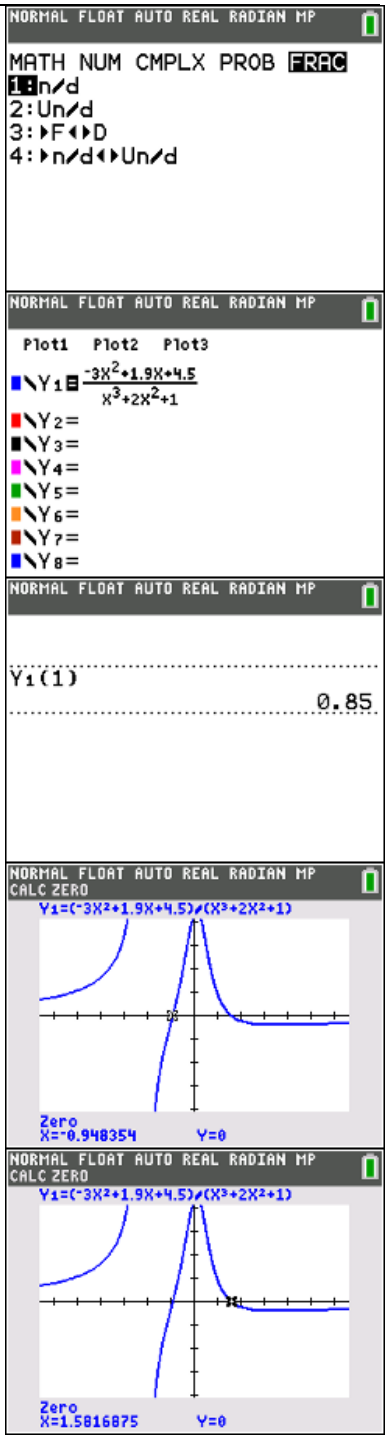
Calculate the value on a calculator screen.

Use a graph of the function to calculator the zeros.

Class Discussion:

Why does this function's values approach zero as x increases without bound?

Possible Answers: If we look at the polynomials in the numerator and denominator, the cubic in the denominator dominates the quadratic in the numerator.



Note: *The following problem 8 is not discussed in the video.*

Problem 8. (A) – (C)

The cost of an Uber ride in Boston is modeled by the function C given by

$$C(m)=\begin{cases} a\cdot m+b\cdot m^2 & \text{if } 0<m\le 5 \\ d\cdot(m-5)+25 & \text{if } m>5 \end{cases}$$

where m is measured in miles and C is measured in dollars. Two Uber riders reported that for $m=1$ the cost was \$9.00 and for $m=3$ the cost was \$21.00.

(A) (i) Use the given data to write two equations that can be used to find the values for the constants a and b in the expression for $C(m)$.

(ii) Find the values of constants a and b.

(B) (i) Use the given data to find the average rate of change of the cost of a ride, in dollars per mile, from $m=2$ to $m=4$. Show the computations that lead to your answer.

(ii) Interpret the meaning of your answer from (i) in the context of the problem.

(iii) The two pieces of the function C connect at the transition point when $m=5$. It is known that $\lim_{m\to 5} C(m)=25$ and $C(6)=27.5$. Consider the average rates of change of C from $m=5$ to $m=p$ miles, where $p>5$. Are these average rates of change less than or greater than the average rate of change from $m=2$ to $m=4$ miles found in (i)? Explain your reasoning.

(C) (i) Using the model C to predict the cost of an Uber ride, what is the maximum amount a rider could pay? Explain your reasoning.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The tasks that students need to be able to perform using the TI-84 for the exam.

For more videos from the AP Precalculus Live series, visit our playlist
https://www.youtube.com/playlist?list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI