

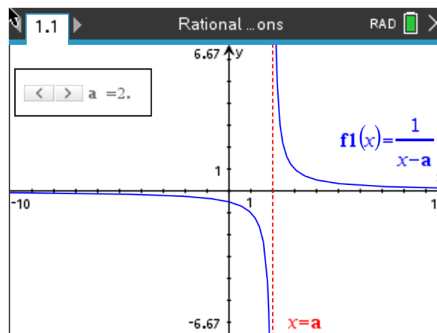


# Rational Functions

## Student Activity

Name \_\_\_\_\_  
Class \_\_\_\_\_

In this activity, you will explore the properties of functions of the forms  $f(x) = \frac{1}{x-a}$  and  $f(x) = \frac{ax+b}{cx+d}$ , and learn to create sliders that allows you to dynamically examine these rational functions generated by quotients of linear functions.

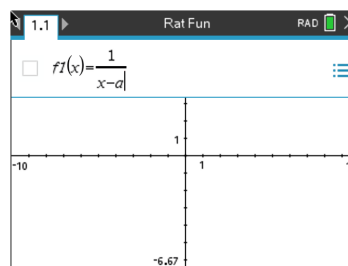


Using the handheld, students will discover, or re-discover, the connection between a rational function, transformations, and both vertical and horizontal asymptotes. To accomplish this, they will be creating and using a dynamic tool called a slider.

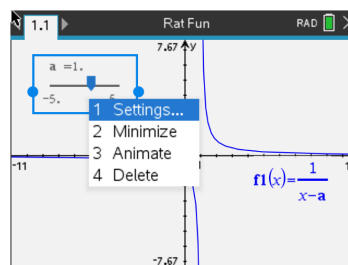
Let's review the idea behind vertical and horizontal asymptotes. A **vertical asymptote** is a vertical line that guides the graph of the function but is not part of it. It can never be crossed by the graph because it occurs at the x-value that is not in the domain of the function. A function may have more than one vertical asymptote. A **horizontal asymptote** is a horizontal line that the graph of a function approaches as the magnitude of the input increases without bound in either a positive or negative direction. A function may cross a horizontal asymptote for finite values of the input.

### Problem 1 – Testing the Slider with Vertical Asymptotes

From the home screen on the handheld, press **1 New** (Document) and press **2 Add Graphs**. In the **f1(x) =**, type the fraction  $\frac{1}{x-a}$ . Once you press **enter** a box will pop up and ask if you would like to create a slider for the variable **a**, press **ok**.



Now that the slider has appeared, you can customize it to your liking. The default slider has a minimum **a** value of -5 and a maximum of 5. It also contains an arrow that you can manipulate to change the **a** value. If you would like to change anything about the slider, press **ctrl menu** to alter





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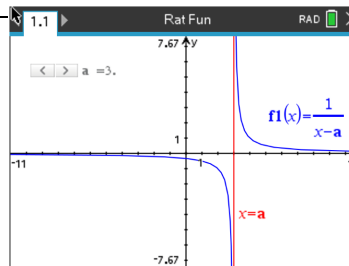
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Let's add one more piece to the slider puzzle to really enhance the connection to asymptotes.

While on the **Graphs** page, press **menu**, **3 Graph Entry/Edit**, **3 Equation Templates**, **1 Line**, **2 Vertical Line  $x = c$** . Type **a** into the template box and press **enter**.



### Discussion Questions

the settings, minimize to a button slider, or animate the slider.

1. With a classmate, press or move the slider and discuss what you see. Write an explanation and share with the class what you notice about the relationships between the rational function and the vertical line.

2. Find the value of  $x$  where the function  $f_1(x) = \frac{1}{x-2}$  is undefined.

3. Find the value of  $x$  where the function  $f_1(x) = \frac{1}{x+1}$  is undefined.

4. Find the value of  $x$  where the function  $f_1(x) = \frac{1}{x-a}$  is undefined.

5. Find the value of  $x$  where the function  $f_1(x) = \frac{1}{x-a}$  has a vertical asymptote. With a classmate, come up with a rule to find the vertical asymptote.

Extension Question:

6. Discuss with a classmate what you notice about what happens horizontally as you move the slider left and right. Write an explanation and share with the class.

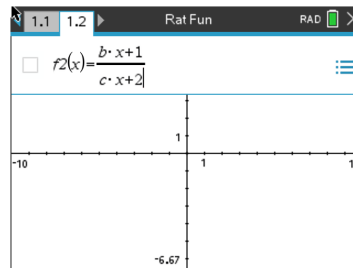


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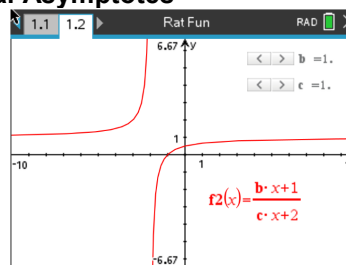
Name \_\_\_\_\_  
Class \_\_\_\_\_

Let's repeat the procedure from **Problem 1**, but this time we will add two sliders. Press **ctrl, doc** and add another **Graphs** page. Enter the fraction  $\frac{b \cdot x + 1}{c \cdot x + 2}$  into **f2(x) =** and press **enter**.  
When the box pops up and asks if you would like to create a slider for the variables **b** and **c**, press **ok**.

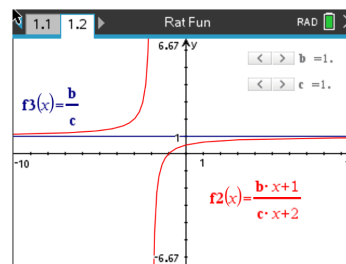


### Problem 2 – Using Sliders to Make Connections to Horizontal Asymptotes

Now that the slider has appeared, you can customize it to your liking, by pressing **ctrl menu**. The picture at the right shows minimized sliders and they have been moved to the first quadrant.



To make the connection once again to asymptotes, we will add a line. Press **tab** and enter the fraction  $\frac{b}{c}$  into **f3(x) =** and press **enter**.



### Discussion Questions

1. With a classmate, press or move the sliders and discuss what you see. Write an explanation and share with the class what you notice about the relationships between the rational function and the horizontal line.
2. Find the value of  $f2(x)$  that the function seems to approach as the input values approach both positive and negative infinity when  $f2(x) = \frac{2x+1}{3x+2}$ .



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3. Find the value of  $f_2(x)$  that the function seems to approach as the input values approach both positive and negative infinity when  $2(x) = \frac{4x+1}{x+2}$ .

4. Find the value of  $f_2(x)$  that the function seems to approach as the input values approach both positive and negative infinity when  $2(x) = \frac{bx+1}{cx+2}$ .

5. Find the value of  $f_2(x)$  that the function  $f_2(x) = \frac{bx+1}{cx+2}$  has a horizontal asymptote. With a classmate, come up with a rule to find the horizontal asymptote.

### Further IB Application

The function  $f$  is defined by  $f(x) = \frac{8x-8}{3x-6}$  for  $x \in \mathbb{R}, x \neq 2$ .

(a) Find the zero of  $f(x)$ .

(b) For the graph of  $y = f(x)$ , write down the equation of

(i) the Vertical Asymptote;

(ii) the Horizontal Asymptote.

(c) (i) Find  $f^{-1}(x)$ , the inverse function of  $f(x)$ .

(ii) Hence, or otherwise, find the value of  $f^{-1}(0)$ .