Points on a Line

Math Objectives

- Students will recognize that the ratio of vertical change to horizontal change is constant between any two distinct points on a line.
- Students will recognize that different lines can have the same or different ratios of vertical change to horizontal change.

Vocabulary

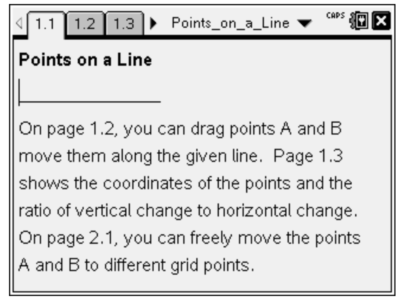
- coordinates
- horizontal
- ratio
- vertical

About the Lesson

- The *Points on a Line* activity is intended to develop a foundation for student understanding of the *slope of a line*. The activity is based on investigating the vertical change and the horizontal change when moving between two points on a line.
- As a result, students will:
  - Move points, and observe the changes in the vertical and horizontal moves to get from one point to another.
  - Create a ratio of vertical change to horizontal change.
- The word slope is not used in the activity but is developed in the activity Understanding Slope.

TI-Nspire™ Navigator™ System

- Use Live Presenter to discuss vertical and horizontal change.
- Use Quick Poll to discuss students' responses and assess students' understanding.
- Use Screen Capture to identify and discuss patterns.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity
Points_on_a_Line_Student.pdf
Points_on_a_Line_Student.doc

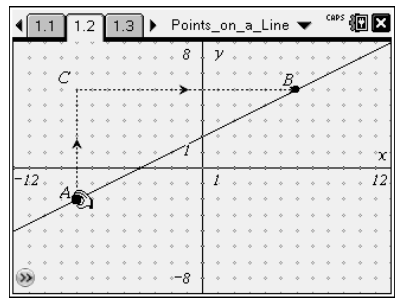
TI-Nspire document
Points_on_a_Line.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty moving a point, move the cursor to hover over the point until a pair of horizontal and vertical arrows appears. The right, left, up and down directional arrows on the touch pad will move the point on grid points. Students might also drag the point. Check to make sure that they have moved the cursor (arrow) until it becomes a hand (☝) getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text*. Then press ctrl ☝ to grab the point and close the hand (✊). When finished moving the point, press esc to release the point.

Move to page 1.2.



1. Describe the vertical and horizontal moves you would make to get from point *A* to point *B*.

   Answer: You would move up 7 units and to the right 14 units.

Teacher Tip: Students may say "positive seven," which is good as a precursor to question 5 below.

TI-Nspire Navigator Opportunity: ***Live Presenter***
See Note 1 at the end of this lesson.

2. a. Move point *A* until the vertical path from point *A* to point *C* is up 2 spaces. Describe the horizontal move to get to point B from point C.

   Answer: The horizontal move would be 4 units to the right.

   b. Move point *A* until the vertical path is up 3 spaces. How much is the horizontal move to get to point *B*?

   Answer: To move from *C* to *B*, I had to move to the right 6 spaces.

Teacher Tip: Students should realize that when moving from one point to another, the vertical and horizontal moves involve direction as well as distance. Be sure students remember that the quadrants are labeled from 1 to 4 counterclockwise – beginning with the first quadrant where *x* and *y* are both positive.

c. What horizontal move will correspond to a vertical move of up 6? Move the point to check your answer.

Answer: To move from *C* to *B*, I had to move to the right 12 spaces.

Teacher Tip: Be sure that students are describing both the distance and the direction.

TI-Nspire Navigator Opportunity: ***Quick Poll (Open Response)***
See Note 2 at the end of this lesson.

3. Make a conjecture about the relationship between the number of units and direction from *A* to *C* and *C* to *B*. Choose some new points for *A* and *B*, and verify your conjecture.

Sample Answers: The number of units from *A* to *C* is half the number of units from *C* to *B* (or the number of units from *C* to *B* is double the number of units from *A* to *C*). Students might note that if the direction from *A* to *C* is up, then the direction from *C* to *B* is right. If the direction from *A* to *C* is down, then the direction from *C* to *B* is left.

Teacher Tip: If students move the points so they coincide, ctrl Z will undo the move so the two points are once again distinct.

TI-Nspire Navigator Opportunity: ***Quick Poll (Open Response)***
See Note 3 at the end of this lesson.

4. What happens when point *A* is above and to the right of point *B*? Try several points for which this is true. Do the results support your conjecture? Why or why not?

Sample Answers: The vertical change is down, and the horizontal change is to the left. I counted the spaces and observed the direction of the arrows.

5. Using correct signs, find the ratio of vertical change to horizontal change for several pairs of points on the line. What do you observe about the ratios?

Sample Answers: The ratios are equivalent and are all ½.

> **TI-Nspire Navigator Opportunity:** ***Quick Poll (Open Response)***
> **See Note 4 at the end of this lesson.**

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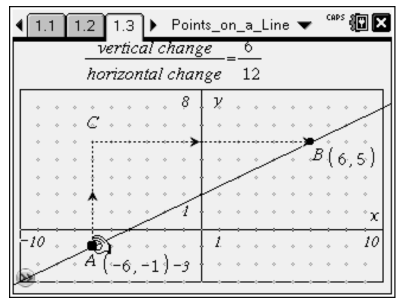


6. Move points *A* and *B* to fill in the missing information in each line of the table below. Explain your reasoning.

Answer: See table below.

	Coordinates of Point A	Coordinates of Point B	Vertical Change (A to C) / Horizontal Change (C to B)
1	(-8 , **-2**)	(**6** , 5)	$\frac{7}{14}$
2	(-6 , **-1**)	(**-2** , **1**)	$\frac{2}{4}$
3	(**2** , 3)	(**8** , **6**)	$\frac{3}{6}$
4	(6 , **5**)	(**-6 , -1**)	$\frac{-6}{-12}$

Teacher Tip: Students might struggle with the fact that a ratio with two negative signs is the same as the ratio where both numerator and denominator are positive. Some of them might also be subtracting the values to find the vertical and horizontal changes, but depending on the order in which they subtract, they might get both positive or negative values. Be sure to stress that if they do this, they have to start from the same point to calculate both the horizontal and vertical change.

7. Describe how the information in the table in question 6 relates to your observations in question 5.

   Answer: The ratios are equivalent and are all ½.

8. Suppose points *A* and *B* are on the line but not displayed in the window of the document. If the vertical change from point A to point B is 50, what is the horizontal change? Explain your reasoning.

   Sample Answers: The horizontal change would be 100. The ratio has to remain ½, so if the numerator is 50, the denominator has to be 100.

   > **Teacher Tip:** The vertical change of 50 means "up 50" or "positive 50." Ask students to consider what if the vertical change is "negative 50"? What if the horizontal change is 50?

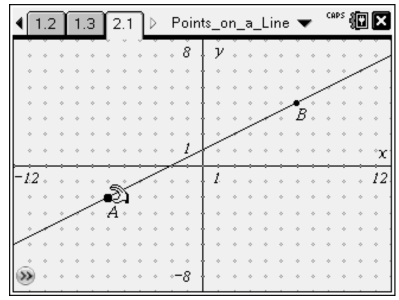
TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*
See Note 5 at the end of this lesson.

9. For a different line, the coordinates of point A are (-3, -4), and the ratio of the vertical change to the horizontal change is equivalent to $\frac{2}{3}$. Find the coordinates of another point on the line. Explain your reasoning.

   Sample Answers: Many different responses are possible. (0, -2), (3, 0), (6, 2). "I moved up 2 and right 3."

   > **Teacher Tip:** Students should be able to show how their points are generated using the ratio 2/3. Sharing responses will help them realize that the ratio of the vertical change to the horizontal change for any two points on a line is constant. You might want to use page 2.1 of the TI-Nspire document. This page allows points *A* and *B* to be freely moved on the grid so that many different lines can be displayed.

Move to page 2.1.



10. Place points *A* and *B* so the move from point A to point B could be described as "down 4 and right 2." Give the coordinates of your points.

    Sample Answers: Point A would have to be to the left of, and above, Point B. The line goes downward from left to right. The ratio for the slope triangle would be negative. There are an infinite number of lines that can have this same ratio, but without a starting point, you cannot determine the exact line.

    > **Teacher Tip:** The lines with the same ratio or slope are said to be parallel. Point out to students that to determine a line you need either two points or a point and the vertical and horizontal moves to get to a second point.

11. a. Siree chose two points on a line, and Paul chose two other points on the same line. Could they get different ratios for the vertical change to the horizontal change? Why or why not?

    Sample Answers: If they both choose points on the same line, they could have different horizontal and vertical changes, but the ratios would have to be equivalent. (As fractions, the ratios will reduce to the same number.)

    > **Teacher Tip:** Another way to think about this is that the triangles formed by the vertical and horizontal changes would all have to be similar in order for the points to be on the same line.

    b. Tamera and Sam each have a line. Tamera chooses two points on her line, and Sam chooses two points on his. They find they have the same ratio of vertical change to horizontal change. Did their lines have to be the same? Why or why not?

    Sample Answers: They could have the same ratios but not necessarily the same line. Tamera could start at a point say (4, 3) and go up one unit and to the right two units to obtain the point (6, 4) and that would produce a line through those two points. Sam could have started at the point (4, 5) and use the same ratio of ½ to make the line through (6, 6), which would be above and parallel to Tamera's line.

TI-Nspire Navigator Opportunity: *Quick Poll (Yes/No)*
See Note 6 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to:

- Describe the vertical and horizontal movement using both direction and distance from one point to another on a line.
- Understand that the ratio of vertical change to horizontal change is constant between any two points on a line.

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Note 1

Question 1, *Live Presenter*: Consider introducing the file using Live Presenter. To clarify the difference between horizontal and vertical moves, ask the student presenter to identify the vertical and horizontal legs of the triangle.

Note 2

Question 2, *Quick Poll (Open Response)*: Send three Open Response Quick Polls, asking students to submit their responses to questions 2a, 2b, and 2c. Explain the format you would like students to follow in answering the question. For example, "First, tell me how many units you must move. Next, tell me the direction, using *L* for *left*, *R* for *right*, *U* for *up*, and *D* for *down*. For example, if you are moving 3 units up, please input 3U with no space." If you are not specific, you will get answers in many different forms and will have to scroll through a large number of responses. By specifying the form of the answers, you will only have to go through a limited number of responses. If a large number of students struggle to identify the correct distance or direction, use Live Presenter to clarify the correct responses.

Note 3

Question 3, *Quick Poll (Open Response)*: Ask students to summarize their conjectures using one word, and then ask them to submit this word using an Open Response Quick Poll. The most common correct answers will be "half" and "double," since the distance from *A* to *C* is half the distance from *C* to *B* (and the distance from *C* to *B* is double the distance from *A* to *C*). Using Live Presenter, ask a student who submitted "half" to explain his or her conjecture. Then ask a student who submitted "double" to explain his or her conjecture.

Note 4
Question 5, *Quick Poll (Open Response)*: Send an Open Response Quick Poll, asking students to submit their answers to question 5. Students can respond with ½ or any equivalent fraction. Ask students to compare the responses and discuss how they are alike or how they are different. Consider taking a Screen Capture, which will result in a variety of different vertical and horizontal distances. Discuss the relationship between these distances, the ratio ½, and the similarity of the various triangles.

One common problem is that students may not understand the word “ratio.” Probe to see if students understand what is meant by *ratio*. If a few students chose ratios that are not equivalent to ½, consider asking them to explain their thinking, either individually or as a class. Often students will find their own errors in trying to explain their answers. Students should recognize that a ratio with two negatives is really a positive, and the overall description of how the line changes from point to point is by a ratio of 1 to 2. Students should also recognize that the line rises from the bottom left to the top right of the graph.

Note 5
Question 8, *Quick Poll (Open Response)*: Send an Open Response Quick Poll, asking students to submit their responses to question 10. Since students cannot move points *A* and *B* to produce a vertical change of 50, they need to reason out their answer. This may be difficult for some of them. If you find that many students were unable to obtain the correct answer to question 10, give them different values for a vertical or horizontal change to help them understand this process.

Note 6
Question 11, *Quick Poll (Yes/No)*: Send a Yes/No Quick Poll, asking students to submit their responses to question 11a. In this case, both “yes” and “no” could be correct depending on students’ reasoning. Ask a student who chose “yes” to explain his or her reasoning. The student should indentify that Siree and Paul could have different horizontal and vertical changes, resulting in different ratios. Then ask a student who chose “no” to explain his or her reasoning. The student should identify that even though Siree and Paul may have different horizontal and vertical changes, both of their ratios can be reduced to ½.

Send an additional Yes/No Quick Poll, asking students to submit their responses to question 11b. Students should submit “no” since Tamera and Sam do not necessarily have to have the same line. Ask a student who responded “no” to explain his or her reasoning. Consider using the word “parallel” to describe the relationship between the two lines.