

Approximate Areas

Student Worksheet

7 8 9 10 11 12



TI-30XPlus
MathPrint™



Worksheet



Student



45 min

Choosing a shape

The area bounded by the curve $f(x) = x^2$, the x axis and the line $x = 4$ is shown opposite.

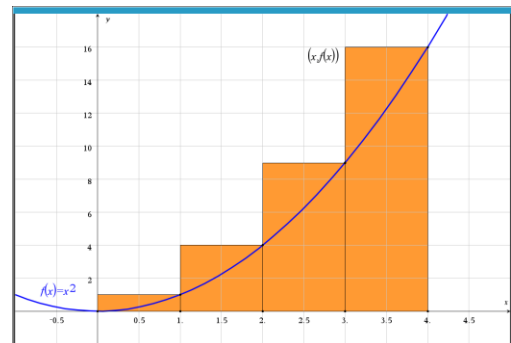
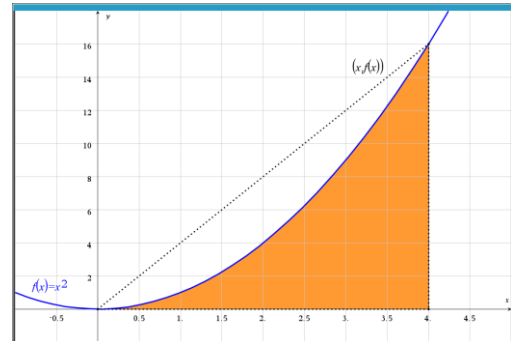
Question: 1.

The area can be approximated by a single triangle with vertices: $(0, 0)$, $(0, 4)$ and $(4, 16)$. Determine the area of the triangle.

Question: 2.

Another way to estimate the area is to use a series of equally spaced rectangles. (Shown opposite)

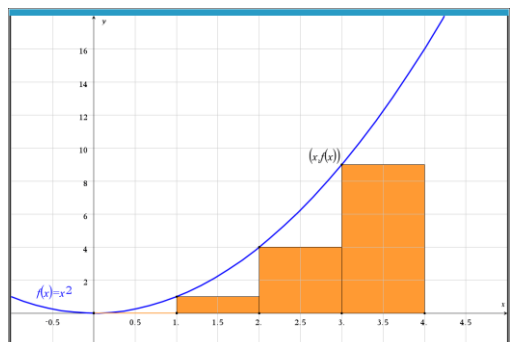
- What is the width of each rectangle?
- The height of each rectangle is defined by the point where the top right corner intersects the curve. Determine the height of each rectangle and hence the approximate area.
- Which approximation, triangle or rectangle, is closest to the actual area? (Explain).
- Would your answer to the previous question (Part iii) be the same if the area was being estimated between $x = 0$ and $x = 3$ using three right bound rectangles versus a single triangle?



Question: 3.

The rectangles shown opposite are referred to as “left bound”. The height of each rectangle is defined by the upper-left corner.

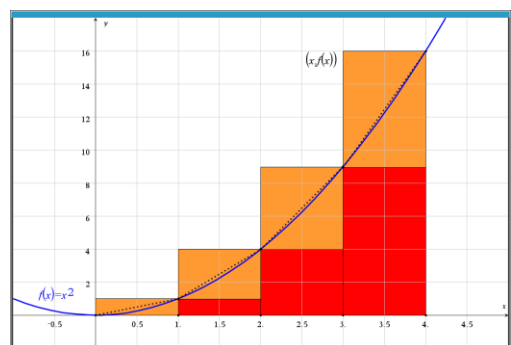
- How will this area estimate compare with previous estimates and the actual area?
- Determine the height of each rectangle and hence the approximate area according to the left bound rectangles.



Question: 4.

The graph opposite shows both the left and right bound rectangles. Averaging the two areas will provide a better approximation. Dotted line segments have been included to illustrate how this averaging may be visualised.

- What shapes are formed geometrically by the averaging of the left and right rectangles?
- Determine the average of the left and right bound rectangles and comment on the result.



The calculations required for three regions can be done quite easily and quickly by hand, if more divisions are created the calculations become repetitive and time consuming! The instructions below demonstrate how to perform these types of calculations efficiently on your calculator.

You can also scan the QR code to watch a video tutorial!



Using your calculator – Lists

The first step is to enter the function. Press:

expr-eval

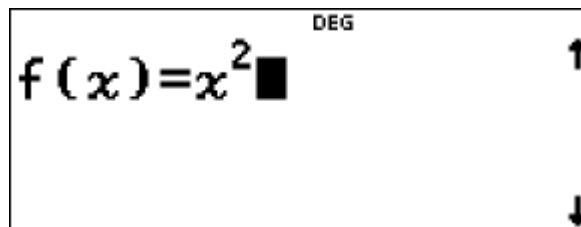
table

... select **Add/Edit Function**

Enter the function $f(x) = x^2$ and then press:

enter

... now quit from the function menu.



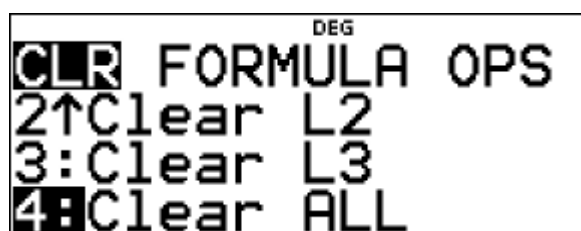
The calculator's lists can be used to perform multiple calculations quickly and efficiently. Make sure the lists are cleared and ready for the calculations.

stat-reg/distr stat-reg/distr

data

data

... clear lists as necessary.



The next step is to generate a list of the x values that form the boundary conditions. For small lists, the entries can be done manually, for larger lists, use the sequence command.

stat-reg/distr

data

navigate to **OPS** and select **Sequence**

The sequence of numbers will go in List 1 (L1)



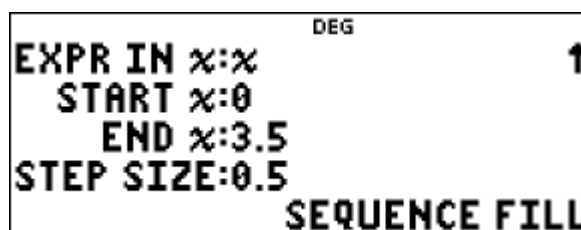
The sequence formula is simply: x

To produce left bound rectangles of width 0.5, the x-values need to be: 0, 0.5, 1, 1.5, 2, 2.5, 3 and 3.5

Enter the settings as shown opposite to produce this result, then press:

enter

...to generate the sequence (list).



Navigate across to List 2 (L2). Then press:

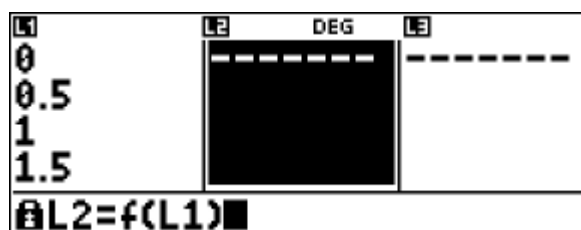
stat-reg/distr

data

... select **Formula** then **Add/Edit Formula**

The **Table** key can be used to access the functions and the **Data** key to access list names.

Match the formula shown opposite for the values in List 2.

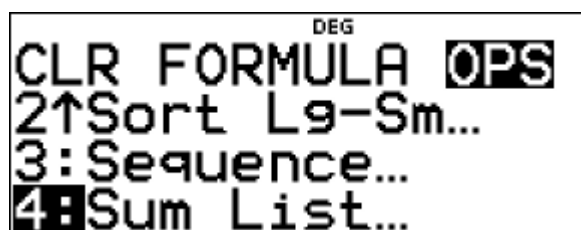


To sum all the values in List 2, press:

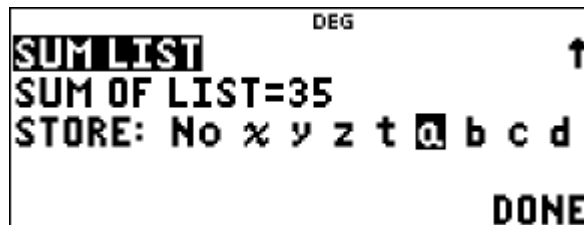
stat-reg/distr

data

... select **Ops** then **Sum List**.



Make sure List 2 is selected for the sum.
The sum is displayed on screen (35 in this example) and can be stored in one of the calculator memories.
Store the result in 'a'.



Recall that the columns in this example are only 0.5 units wide, so the area is equal to: $a \div 2$.



Question: 5.

Use your calculator to determine the approximate area for the same function and region from the previous question, using a column width of 0.5 units for:

- i) Left Bound rectangles
- ii) Right Bound rectangles
- iii) Trapeziums

Question: 6.

Use your calculator to determine the approximate area for the same function and region from the previous question, using a column width of 0.1 units for:

- i) Left Bound rectangles
- ii) Right Bound rectangles
- iv) Trapeziums

Question: 7.

Use your calculator to determine the approximate area bounded by the curve: $y = 4 - x^2$ and the x axis using a column width of 0.1 units for:

- i) Left Bound rectangles
- ii) Right Bound rectangles
- iii) Explain why the left and right bound rectangle calculations are the same.
- iv) Determine the approximate area using trapeziums of width 0.1 units.

Question: 8.

Use your calculator to determine the approximate area bounded by the curve: $y = 4 - x^2$, the y and x axes where $x \geq 0$, using a column width of 0.1 units for:

- i) Left Bound rectangles
- ii) Right Bound rectangles
- iii) Explain why the left bound area approximation is larger than the right bound.

Extension – Alternative Approximations

Another approach to approximating the area under a curve is to use the **midpoint rule**. The left and right bound rectangles use the bordering values according to the abscissa (x coordinate), the midpoint rule determines the height of each rectangle based on the midpoint of the abscissa values. For example, finding the approximate area under the curve $y = x^2$ using four intervals, rectangle heights were determined using either left bound: $\{f(0), f(1), f(2), f(3)\}$ or right bound: $\{f(1), f(2), f(3), f(4)\}$. For the midpoint rule, heights are determined using: $\{f(0.5), f(1.5), f(2.5), f(3.5)\}$.

Question: 9.

For the following question, the area being explored is bounded by the curve: $y = x^2$, the x axis and the line $x = 4$.

- Use the midpoint rule to determine the approximate area using a column width of 1 unit.
- Use the midpoint rule to determine the approximate area using a column width of 0.1 units.
- Explain how the midpoint rule is different to the trapezoidal rule.

Another approach to approximating areas is to use **Simpson's rule**. This rule is quite different:

$$S_n = \frac{\Delta x}{3} (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{n-2}) + 2f(x_{n-1}) + f(x_n))$$

To perform this calculation 'automatically' using the lists requires a somewhat complicated formula. The first and last terms: $f(x_0)$ and $f(x_n)$ are left out of the automated calculations. This means the only thing that needs to be sorted is multiplying by 4 for odd terms and 2 for even terms. To achieve this List 1 will be used to store the term number, List 2 for the abscissa and List 3 for the evaluation of $f(x)$.

List 1: Sequence: 1, 2, ... 39

List 2: Sequence: 0.1, 0.2, ... 3.9

List 3: Formula: $4 \times \text{mod}(L1, 2) \times f(L2) + 2 \times \text{mod}(L1 - 1, 2) \times f(L2)$

The modular arithmetic command: $\text{mod}(\#, \#)$ is available from the Math > Number menu. $\text{Mod}(a, b)$ produces the remainder when a is divided by b , so $\text{mod}(1, 2)$, $\text{mod}(3, 2)$, $\text{mod}(5, 2)$... each produce a result of 1.

In contrast, $\text{mod}(2, 2)$, $\text{mod}(4, 2)$, $\text{mod}(6, 2)$... each produce a result of 0. The overall result of the formula is that when List 1 is odd, the result will be $4 \times f(L2)$ and when List 1 is even the result will be $2 \times f(L2)$.

Once the sum of List 3 has been calculated, $f(x_0)$ and $f(x_n)$ can be added to the result and then multiplied by the appropriate fraction to compute the area estimate.

Question: 10.

For the following question, the area being explored is bounded by the curve: $y = x^2$, the x axis and the line $x = 4$.

- Use the table to help compute the approximate area using Simpson's rule with a column width of 0.5 units.

n	0	1	2	3	4	5	6	7	8
x	0	0.5	1	1.5	2	2.5	3	3.5	4
$f(x_n)$	0	0.25							

- Use Simpson's rule to determine the approximate area using a column width of 0.1 units.