Graphs of the OTHER Trig Functions

Math Objectives

- Students will identify the values of cotangent, secant and cosecant for various values of θ from the unit circle.
- Students will understand the correspondence of the graphs of cotangent, secant and cosecant to the unit circle.
- Students will identify and explain features of the graphs of cotangent, secant, and cosecant, where the functions are undefined, or where they have intercepts.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

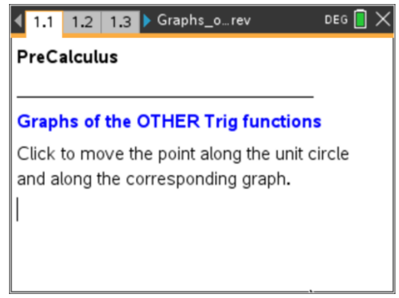
- sine
- cosine
- cotangent
- secant
- cosecant
- unit circle

About the Lesson

- This lesson involves providing opportunities for students to explore and make sense of the graphs of the cotangent, secant, and cosecant functions.
- As a result, students will:
  - Use clickers to move a point around the unit circle, observing the changes in function values for cotangent, secant, and cosecant, and observing the connections to the graphs of each of the three functions.
  - Make and test conjectures about the graphs of the cotangent, secant, and cosecant functions based on observations from the unit circle.

TI-Nspire™ Navigator™ System

- Use Quick Poll to assess student understanding.
- Use Screen Capture to display correspondences between graphs of sine and cosine and each of the three trigonometric functions featured in this activity.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity
Graphs_of_the_OTHER_Trig_Functions_Student.pdf
Graphs_of_the_OTHER_Trig_Functions_Student.doc

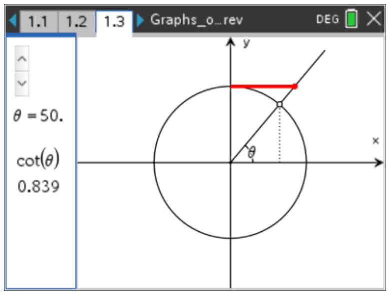
TI-Nspire document
Graphs_of_the_OTHER_Trig_Functions.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: Click the sliders to change the value of θ and move the point around the unit circle.

Move to page 1.3.



1. The screen shows a unit circle, and you can use the clicker to move a point around the circle, changing the angle θ. Change the angle θ to be equal to 35°, as pictured on the right. This creates two right triangles: The one on the top with one leg on the y-axis, one leg the bolded line segment, and the one on the bottom with one leg on the x-axis and one leg the dashed line segment.
   a. Prove that these two triangles are similar.

   Answer: Let the angle adjacent to θ in the interior of the upper triangle be α. Then $\alpha + \theta = 90°$, so the other angle in the upper triangle must be the same in measure as θ. Thus we have two right triangles with one angle measuring θ; therefore, the triangles are similar by the AA similarity condition.

   b. Using the ratios of the non-hypotenuse side lengths, determine the length of the bolded side in terms of the other legs of the triangles. Don't forget that you are working on a unit circle!

   Answer: Let the horizontal and vertical sides of the lower triangle be *a* and *b*, respectively. Let the horizontal and vertical sides of the upper triangle be *c* and *d*, respectively. Then, by similarity of the two triangles, we have $\frac{c}{d} = \frac{a}{b}$. Then the bolded side of the upper triangle, its horizontal leg, is $c = d \cdot \frac{a}{b}$. But the vertical leg *d* is a radius of the unit circle, so *d* = 1. Thus $c = \frac{a}{b}$.

TI-Nspire Navigator Opportunity: ***Quick Poll (Open Response)***
See Note 1 at the end of this lesson.

Teacher Tip: Making this connection is critical, and might require additional discussion. Students must understand that the bold line segment is equal in length to the ratio *a*/*b*. This value is the reciprocal of the value of the tangent of θ, or the cotangent of θ. It will be important that students make the important connection. Similarly, on each of the secant and cosecant, you should check in with students to ensure that they understand WHY the bold line segment length in each case represents the value of that trigonometric function at the given angle.

2. The screen shows a unit circle, and you can use the clicker to move a point around the circle, changing the angle θ.

   a. Recall that we can write the trig functions in terms of the sine and cosine functions. What is the cotangent function in terms of the sine and cosine functions?

   Answer: $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

   Teacher Tip: Be prepared to address sloppy notation. Cotangent, sine, and cosine are all functions, requiring arguments. Therefore, the response $\cot = \dfrac{\cos}{\sin}$ is incorrect.

   b. Click to move the point around the circle. What does the bold line segment length represent? How is it determined? How do you know?

   Answer: The length of the bold line segment is the value of $\cot\theta$ at that value of θ. It is determined from $\dfrac{\cos\theta}{\sin\theta}$. From question 1, the bold line segment length is the ratio *a*/*b*, which is the reciprocal of the tangent of θ, or the cotangent of θ.

   c. For what values of θ is the cotangent 0? Undefined? Why?

   Answer: $\cot\theta = 0$ when $\cos\theta = 0$, thus for $\theta = 180k + 90$. $\cot\theta$ is undefined when $\sin\theta = 0$, thus for $\theta = 180k$.

TI-Nspire Navigator Opportunity: ***Quick Poll (Open Response)***
See Note 2 at the end of this lesson.

c. Describe or sketch what you think the graph of the cotangent function might look like. Explain your thinking.

Sample Answer: The cotangent function will be very large near 0°, 0 at 90°, and a very large negative value close to 180°. It is periodic.

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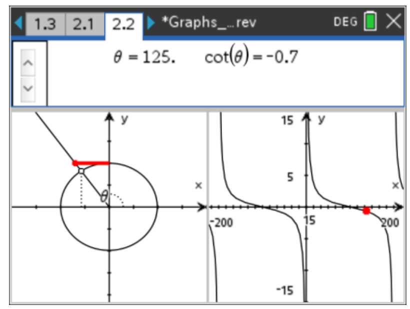

3. The left hand side of the screen shows the unit circle from Page 1.3, and the right side shows the graph of the cotangent function.
   a. How does the graph of the cotangent function compare to your description in question 1? If your prediction was incorrect, what do you think was your mistake?

   Sample Answers: Student answers will vary. One issue with predictions may be correctly determining the concavity of the graph.

   b. What happens as you click the arrow to change the value of θ? Why does the graph "jump" from one piece to another?

   Answer: The point moves along the graph of the cotangent. It "jumps" when the value of the cotangent is undefined.

   c. Why does the graph repeat itself? As θ increases, how many times do you expect the graph to repeat? Why?

   Answer: The graph repeats itself because the cotangent function is periodic. It will continue infinitely many times because the domain of the cotangent function is infinite.

> **Teacher Tip:** It is important to note that while the screen only shows 2.5 periods of the function, it is defined beyond that.

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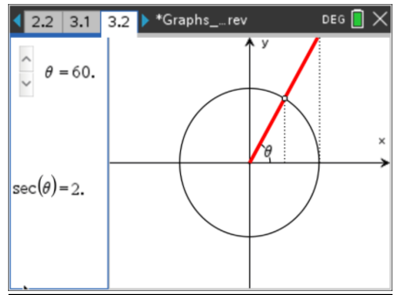

4. a. Recall that we can write the trig functions in terms of the sine and cosine functions. What is the secant function in terms of the sine and cosine functions?

Answer: $\sec\theta = \dfrac{1}{\cos\theta}$

b. Click to move the point around the circle. What does the bold line segment length represent? How is it determined?

Answer: The bold line segment has a length of $\sec\theta$ at that value of θ because It equals $\dfrac{1}{\cos\theta}$. Use similar triangles, and observe that the length of the bold line segment is 1/*a*, where *a* is the length of the horizontal leg of the inner triangle, or $\dfrac{1}{\cos\theta}$.

c. For what values of θ is the secant 0? Undefined? Why?

Answer: $\sec\theta$ is never 0, since it is impossible for a fraction with numerator 1 to be 0. $\sec\theta$ will never be smaller in magnitude than 1, as 1 is the maximum magnitude of $\cos\theta$. It is undefined at values of θ = 180*k* + 90°, where $\cos\theta$ is 0.

d. Describe or sketch what you think the graph of the secant function might look like. Explain your thinking.

Sample Answers: The secant function will be undefined at 180*k*+90°, will attain relative minima of 1 at 360*k*°, and will attain relative maxima of -1 at 180(2*k*+1)°.

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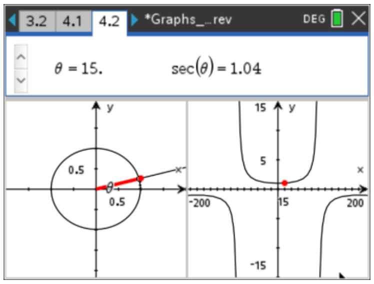

5. The left hand side of the screen shows the unit circle from Page 3.2, and the right side shows the graph of the secant function.

   a. How does the graph of the secant function compare to your description in question 3? If your prediction was incorrect, what do you think was your mistake?

   Sample Answers: Students might have incorrectly predicted the concavity of the graph or the extrema.

   b. What happens as you click the arrow to change the value of θ? Why does the graph "jump" from one piece to another? Why doesn't the graph ever cross the x-axis?

   Answer: The point traces on the graph of the function as well. It "jumps" because there are places where the function is undefined. The graph never crosses the x-axis because the function is never 0.

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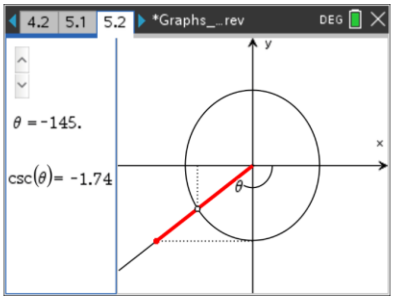

6. a. How can you write the cosecant function in terms of the sine and cosine functions?

   Answer: $\csc\theta = \dfrac{1}{\sin\theta}$

   b. Click to move the point around the circle. What does the bold line segment length represent? How is it determined?

   Answer: The length of the bold line segment is the value of $\csc\theta$ at that value of θ . It is determined from $\dfrac{1}{\sin\theta}$. Using similar triangles again, the length of the bold line segment is equal to 1/*b* where b is the length of the vertical leg of the lower triangle. Of course, *b* = $\sin\theta$, as we are working in a unit circle.

c. For what values of θ is the cosecant 0? Undefined? Why?

Answer: $\csc\theta$ is never 0, since it is impossible for a fraction with numerator 1 to be 0. $\csc\theta$ will never be smaller in magnitude than 1, as 1 is the maximum magnitude of $\sin\theta$. It is undefined at 180k°, as this is where $\sin\theta$ is 0.

d. Describe or sketch what you think the graph of the cosecant function might look like. Explain your thinking.

Sample Answers: The cosecant function will be undefined at 180k°, will attain relative minima of 1 at 360*k* + 90°, and will attain relative maxima of -1 at 180(2*k*+1) + 90°.

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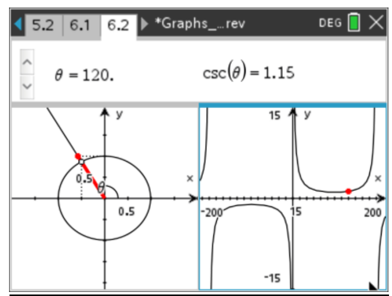

7. How does the graph of the cosecant function compare to your description in question 5? Explain.

Sample Answers: Students might have incorrectly predicted the concavity of the function, or might have failed to correctly predict the extrema.

8. Choose one of the three trig functions from this activity. How could you use the graphs of sine and cosine to graph that function? Explain.

Sample Answers: Plotting the sine, cosine, and cotangent functions together, students might be able to identify the intersection points of sine and cosine as the places where the cotangent is 1, the zeroes of sine as the places where the cotangent is 0, and the zeroes of cosine as the places where the cotangent is undefined.

TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The relationships between the sine and cosine functions and each of the cotangent, secant, and cosecant functions.
- The correspondences between the unit circle and the graphs of each of the cotangent, secant, and cosecant functions.

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Note 1

Question 1b, *Quick Poll (Open Response)*: Use the Quick Poll to determine students' understanding of the length of the bolded line in terms of the sides of the original triangle. This is a critical step to making sense of the use of the graphical representation as the activity progresses.

Note 2

Question 2c, *Quick Poll (Open Response)*: Use the Quick Poll to determine students' understanding of the places where cotangent is 0 or undefined. This is an opportunity to introduce generalization (i.e. beyond [-180, 180]) and to ensure that students are correctly identifying these values.

Note 3

Question 8, *Screen Capture*: Students might open a new graph page and plot sine, cosine, and the other trig function of their choice. Sharing these graphs provides an opportunity to discuss the correspondences between the three graphs, with particular focus on zeroes, extrema, and locations where the functions are undefined.

Extension

Teachers might want to have students explore the results of a phase shift, vertical shift, or a change in amplitude in sine and cosine on the remaining trigonometric functions.

Teachers might want to have students explore using the sine, cosine, or tangent functions to determine the graphs of cosecant, secant, and cotangent, respectively.