

# Transformations of Functions

## Teacher Notes and Answers

7 8 9 10 11 12



### Introduction

Functions can be reflected, dilated or translated, collectively these are called transformations. The purpose of this activity is to focus on the transformations. Data has been collected using a variety of sensors to measure the following:

- Motion of a bouncing ball
- Motion of a pendulum
- Discharge of a capacitor

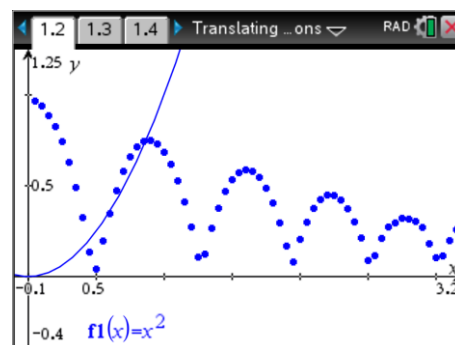
For each of these sets of data, an appropriate function has been defined if  $f_1(x)$ . Transformations to this function can be applied using  $f_2(x)$ . An extension option includes modelling of the Olympic rings. All the data and corresponding images are loaded onto the file: Transformations.tns. Load this file onto your calculator and navigate to page 1.2

### Bouncing Ball

A quadratic function has been defined in  $f_1(x)$ .

Use the tab key to navigate to  $f_2(x)$  and start by entering the following:

$$f_2(x) = -f_1(x)$$



### Question 1

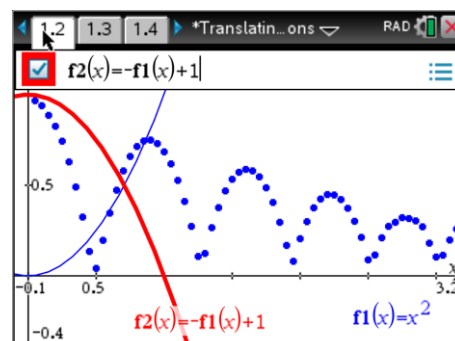
How does this first transformation affect the graph?

**Answer:** The graph is a reflection of the original function in the x axis.

The next step is to provide explore another transformation of the function. Edit the function defined by  $f_2(x) = -f_1(x) + k$  by trying different values of  $k$  with an attempt to model the first 'drop' of the bouncing ball.

#### Note:

It is not essential at this stage to get the ideal value of  $k$ , something relatively close is sufficient as it can be edited again later.



### Question 2

How does this transformation affect the graph?

**Answer:** It is a translation parallel to the y axis.

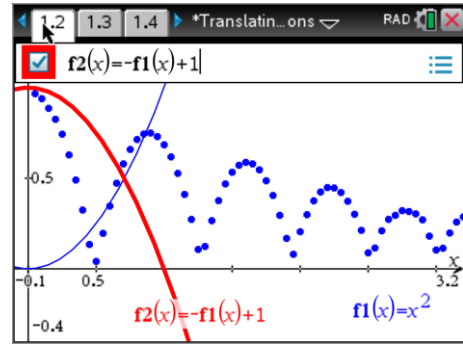
The next step is to provide explore another transformation of the function. Edit the function again, keeping all previous transformation:

$$f_2(x) = -f_1(x - h) + k$$

by trying different values of  $h$ .

**Note:**

Once again, it is not essential at this stage to get the ideal value of  $h$ , something relatively close is sufficient as it can be edited again later.



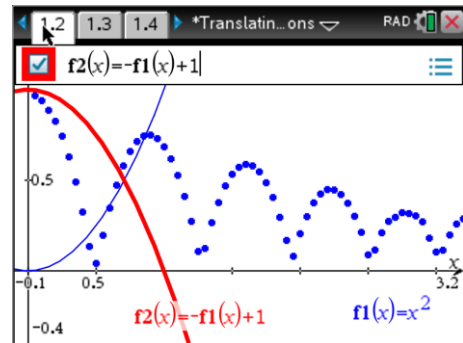
**Question 3**

How does this transformation affect the graph?

**Answer:** It is a translation parallel to the x axis.

The last transformation, keeping all previous transformation is to edit the value of  $a$  in:  $f_2(x) = -af_1(x - h) + k$ .

Once you have explored possible values of  $a$ , edit some of the previous transformations as necessary and try and provide the best match possible for the first drop of the bouncing ball.



**Question 4**

How does this transformation affect the graph?

**Answer:** It is a dilation. Students may say it makes the graph 'wider' or 'narrower', these terms however should be used with caution as the graph is 'infinitely' wide, our view through the window settings however corresponds to the description of 'wider' or 'narrower'.

**Question 5**

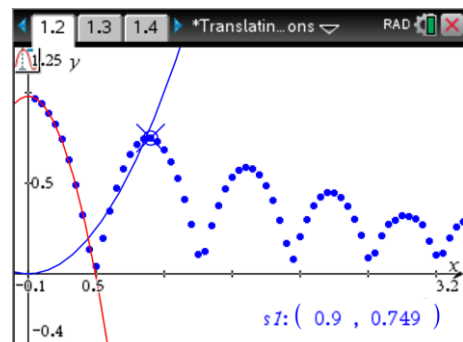
Write down the overall equation for your graph that models the first drop of the bouncing ball?

**Answers will vary, the function:**  $f_2(x) = -4f_1(x) + 0.98$  provides a suitable approximation for the first ball bounce.

The remaining ball bounces can be modelled using the same approach as above.

Use  $f_3(x)$  in terms of  $f_1(x)$  to model the second ball bounce and so on with the remaining ball bounces.

The Graph Trace option is available from the TRACE menu and can be used to determine specific Time and Position coordinates to assist in the transformations. Remember that the data is real so specific points may not represent the ideal location for the turning point on this quadratic function.



**Calculator Tip!**



It can be handy to zoom in on the associated section of the ball bounce to see how close the function models each bounce. The 'zoom box' available from the Window / Zoom menu provides a convenient way of selecting a specific bounce. To return to the complete bounce series either use zoom data (same menu) or write down the minimum and maximum x and y values and use the Window settings option to return to an appropriate view.

**Question 6**

Determine equations for the remaining ball bounces.

**Answer:** Answers will vary, however the value of 'a' in each will be approximately -4.9 (half of gravity) and the translations can be approximated by looking at the peak of each ball bounce using the trace function.

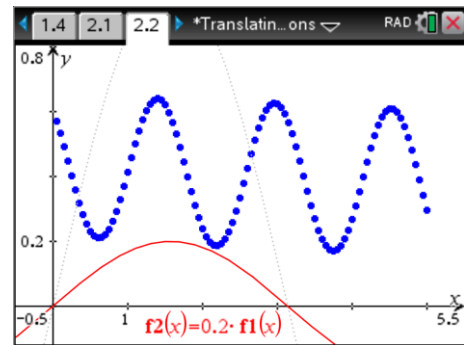
**Pendulum**

Navigate to page 2.2. The motion of a pendulum has been recorded.

A trigonometric function has been defined in:  $f_1(x)$ .

Use the tab key to navigate to  $f_2(x)$  and start by entering the following:

$$f_2(x) = 0.2f_1(x)$$



**Question 7**

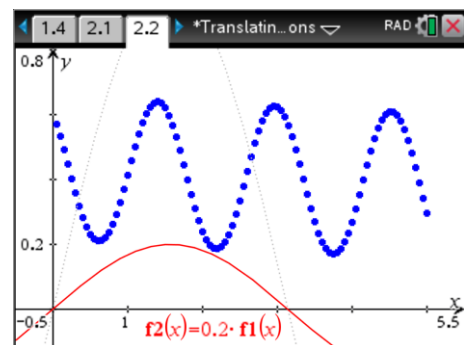
How does this first transformation affect the graph?

**Answer:** Dilation, changes the amplitude of the function. (Dilation parallel to y axis)

The next transformation is of the form:

$$f_2(x) = 0.2f_1(x) + k$$

Explore the values of  $k$  that provide a reasonable transformation. Once again, this is approximate, all these transformations can be edited as the modelling function gets closer to the actual data.



Once you are happy with the previous transformation explore and edit the following transformation:

$$f_2(x) = 0.2f_1(nx) + k$$

Start with a value of  $n = 3$ .

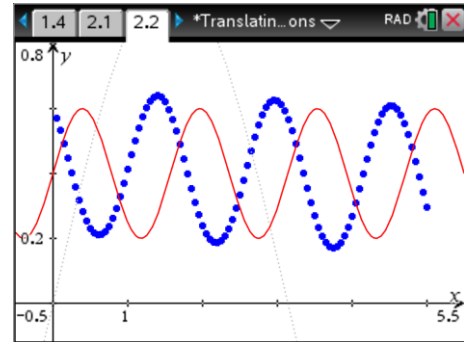
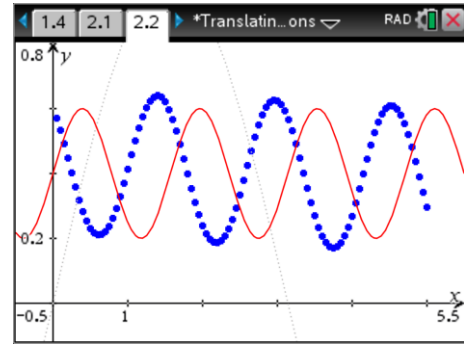
At this stage it is appropriate to edit the previous two transformations.

The final transformation for this function is:

$$f_2(x) = 0.2f_1(n(x-h)) + k$$

Make sure all parentheses are placed appropriately. There are numerous value of  $h$  that will work for modelling the pendulum. Identify one and work on the specific value.

As your function gets closer and closer to the data, edit each parameter accordingly.



**Question 8**

Record the function that best models the pendulum data.

**Answer:** One function that models the data reasonably well:  $f_2(x) = 0.23f_1(4(x-1)) + 0.41$

Note that infinitely many other solutions exist as the function may be translated parallel to the x axis in infinitely many ways. It is also important to note that this is genuine data, so precise modelling is not possible due to the genuinely practical nature of the task.

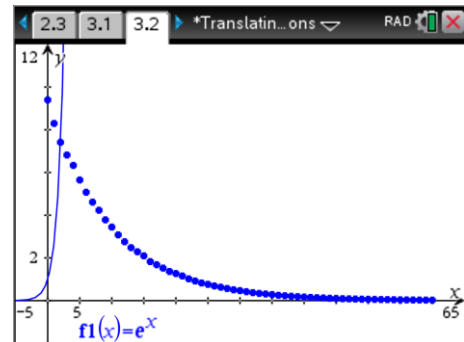
**Capacitor**

Navigate to page 3.2. The electrical discharge of a capacitor has been recorded. A knowledge of electronics is not required for this task, however it is nice to know just how precisely electronics can align to mathematical equations!

An exponential function has been defined in:  $f_1(x)$ .

Use the tab key to navigate to  $f_2(x)$  and explore transformations of the form:

$$f_2(x) = af_1(bx)$$



**Question 9**

Record the function that best models the discharge of a capacitor.

**Answer:**  $f_2(x) = 9.5f_1(-0.1x)$

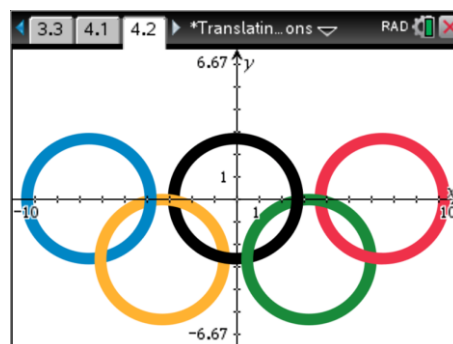
**Extension – Olympic Rings**

Navigate to page 4.2. The Olympic rings have been drawn in the background of the graph. Each ring can be modelled by the relation:

$$(x - h)^2 + (y - k)^2 = r^2$$

This relation can be typed into the calculator by selection “Relation” from the Graph Entry/Edit menu.

Explore each of the transformations:  $h$ ,  $k$  and  $r$ . Note that the colour of the graphs can also be changed to match the rings, however it is best to contrast the colour with the corresponding ring whilst performing the transformations, otherwise the graph can be hard to see on the image.

**Question 10**

Record the five relations that model the Olympic rings.

$$x^2 + y^2 = 2.75^2$$

$$(x + 6.6)^2 + y^2 = 2.75^2$$

Answers:  $(x - 6.6)^2 + y^2 = 2.75^2$

$$(x - 3.3)^2 + (y + 2.7)^2 = 2.75^2$$

$$(x - 3.3)^2 + (y + 2.7)^2 = 2.75^2$$