Using the Document: Taylor_Polynomials_CAS.tns

This calculator file provides a tool for generating and graphing Taylor polynomials. The degree of the Taylor polynomial is changed using the arrow clicker for n, and the value for a can be changed by dragging the point on the x-axis or by entering a new x-coordinate in the ordered pair displayed on the graph screen.

Suggested Applications and Extensions

1. (a) Find the Taylor polynomials up to degree 7 for $f(x) = \sin x$ centered at $a = 0$. Examine these graphs as n increases.

   (b) Evaluate f and these Taylor polynomials at $x = \frac{\pi}{4}, \frac{\pi}{2}$, and π.

   (c) Explain how the Taylor polynomials converge to $f(x)$.

2. Find the Taylor polynomial $T_5(x)$ for the function f centered at the number a. Observe how the graphs of the Taylor polynomials change as n increases, and find an interval in which the Taylor polynomial is a good approximation to f.

   (a) $f(x) = e^x$, $a = -1$

   (b) $f(x) = \cos x$, $a = \frac{\pi}{6}$

   (c) $f(x) = \ln x$, $a = 1$

   (d) $f(x) = x \sin x$, $a = \frac{\pi}{2}$

   (e) $f(x) = x \tan^{-1} x$, $a = -\frac{\pi}{4}$

   (f) $f(x) = x^2 e^{-x}$, $a = \frac{1}{2}$

3. Find the Taylor polynomial $T_5(x)$ for the function f centered at 0. Observed how the graphs change as n increases, find an interval in which the Taylor polynomial is a good approximation to f, and find $T_5(b)$.

   (a) $f(x) = (1 - x)^{-3}$ $b = -\frac{1}{4}$

   (b) $f(x) = \ln(1 + x)$ $b = \frac{1}{2}$

   (c) $f(x) = e^{-x/2}$, $b = 2$

   (d) $f(x) = 3^x$, $b = -\frac{1}{2}$

   (e) $f(x) = x \tan x$, $b = \frac{\pi}{4}$

   (f) $f(x) = \frac{1}{1 + x^2}$, $b = 1$

4. Find the Taylor polynomial $T_5(x)$ for the function $f(x) = x^5 - 3x^3 + x$ centered at $a = 1$. Explain this result.
5. (a) Find the Taylor polynomial $T_3(x)$ for the function $f(x) = e^{x^2}$ centered at $a = 0$.
   (b) Find the Taylor polynomial $T_3(x)$ for the function $g(x) = \ln(x^2 + 1)$ centered at $a = 0$.
   (c) Find the Taylor polynomial $T_3(x)$ for the function $h(x) = e^{x^2} \ln(x^2 + 1)$ centered at $a = 0$. Explain how this Taylor polynomial is related to those found in parts (a) and (b).