Highest Common Factor

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Student Activity

7 8 9 10 11 12

TI-Nspire | Coding | Student | 60 min

Introduction

The Highest Common Factor (HCF), also known as Greatest Common Divisor (GCD) of two numbers is useful for many reasons. A common application relates to fractions, for example:

$$\frac{1}{12}+\frac{1}{18}=\frac{18}{216}+\frac{12}{216}=\frac{30}{216} \qquad \text{OR} \qquad \frac{1}{12}+\frac{1}{18}=\frac{3}{36}+\frac{2}{36}=\frac{5}{36}$$

The left-hand example prioritises a common denominator over highest common factor, the result is a fraction that still needs to be simplified. The right-hand example prioritises a highest common factor and then establishes the common denominator; the result is a fraction in its simplest form. This is just one example where a highest common factor is useful.

More than 2000 years ago, a mathematician by the name of Euclid created an algorithm that helps find the highest common factor of two numbers.

LINE #1: IF A = 0 THEN GCD(A,B) = B since GCD(0,B) = B

LINE #2: IF B = 0 THEN GCD(A,B) = A since GCD(A,0) = A

LINE #3: A = B x Q + R … where Q is the quotient and R is the remainder

LINE #4: GCD(B,R) = GCD(A,B), now find GCD(B,R)

https://bit.ly/EuclidAlgorithm

This algorithm will make more sense when some numbers are used for A and B.
Suppose we want to find the highest common factor of (A) 1260 and (B) 385.
As neither A = 0 or B = 0 we progress to LINE #3.

1260 = 385 x 3 + 105 [We can say that 105 is the remainder when 1260 is divided by 385]

According to LINE #4 of Euclid's algorithm: GCD(1260,385) = GCD(385,105)

We apply the algorithm again as 385 $\neq$ 0 and 105 $\neq$ 0 and proceed to LINE #3.

385 = 105 x 3 + 70 [We can say that 70 is the remainder when 385 is divided by 105]

According to LINE #4 of Euclid's algorithm: GCD(1260,385) = GCD(385,105) = GCD(105,70). As 105 $\neq$ 0 and 70 $\neq$ 0, then we return to LINE #3

105 = 70 x 1 + 35 [We can say that 35 is the remainder when 105 is divided by 70]

We are getting close! According to LINE #4 of Euclid's algorithm: GCD(1260,385) = … = GCD(70,35)

Applying the algorithm one more time, as 70 $\neq$ 0 and 35 $\neq$ 0, we proceed to LINE #3.

70 = 2 x 35 + 0. [This time the remainder is 0!]

Now we can apply LINE #1 or LINE #2 since we have GCD(35,0) = 35.

Our conclusion is that the Highest Common Factor or Greatest Common Divisor of 1286 and 385 is 35.

Question: 1.

Use Euclid's algorithm to identify the highest common factor of: 3850 and 3234.

Author: P. Fox

Creating the Program

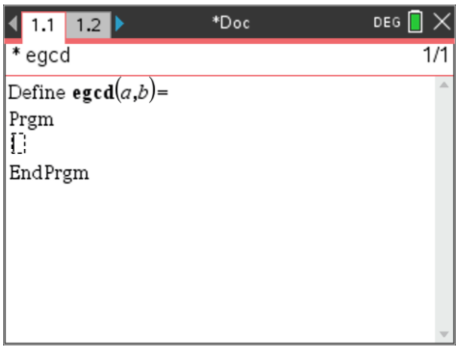
Instructions:

Start a new document; insert a new program.

Add Program Editor > **New**

Call the program: EGCD

Edit the program definition to include “a” and “b”. (See opposite)

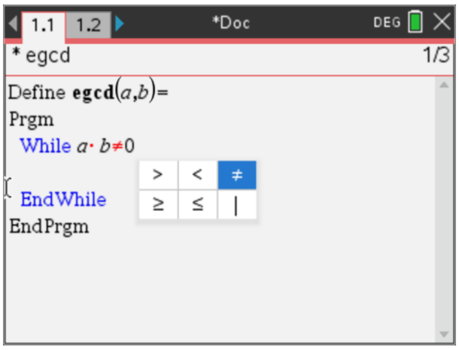


Euclid’s algorithm ceases when either a = 0 or b = 0, an easy way to check this is: a x b = 0. The null factor law states that “if the product of two numbers is zero, then one or both of the numbers must be zero.”

The algorithm should continue to run while a x b ≠ 0.

Menu > **Control** > **While … EndWhile**

The “not equals” sign can be accessed from the inequality flyout menu.

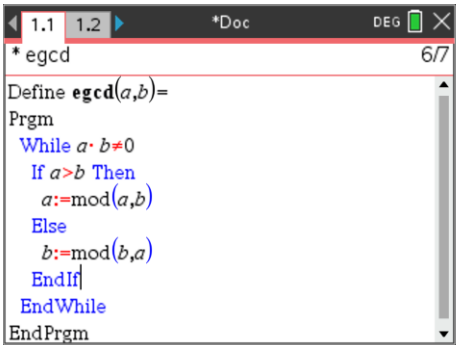


Modular arithmetic returns the remainder when a ÷ b (where a > b) so an If … Then … Else … statement can be used to process Line #3 of Euclid’s algorithm.

Menu > **Control** > **If...Then...Else... EndIf**

The mod() command can be typed directly or accessed from the catalogue. Note carefully the respective orders for a and b.

That’s the entire algorithm! The only thing remaining is to display the results. You can place: Disp a,b in the loop, between EndIf and EndWhile or outside the loop, between EndWhile and EndPrgm.



Question: 2.

What is the difference in the output when the display command (Disp) is placed inside the loop compared with outside? Try it using 1914 and 7293.

Question: 3.

Test your program on some smaller numbers where you know the highest common factor. Record your test results here.

Question: 4.

The **Number** menu in the Calculator Application contains a command to determine the highest common factor of **two** numbers. Adjust your program to find the highest common factor of three numbers or a list of numbers. Example: EGCD(a,b,c) or EGCD({#1,#2 … #n})

Test and evaluate your program.

Author: P.Fox